

Enhancing Hybrid Forecasting Models with Cross-Entropy, Fuzzy-Logic and Attention Mechanisms

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Abstract:

For many decades, time-series forecasting has been applied to different problems by scientists and industries. Many models have been introduced for the purpose of forecasting. Statistical techniques have been applied to this task since many years ago, after which neural network algorithms were introduced. Today, hybrid techniques are gaining popularity, aiming to put together the advantages of these two approaches. These hybrid approaches can provide better forecasting, and at the same time, they can develop a more sophisticated set of visualization analytics for decision support. And recently, the application of cross-entropy, fuzzy logic, and attention mechanisms in hybrid forecasting makes the modeling performed by these advanced systems more capable of modeling complex and uncertain situations in financial markets as well as in the energy market domain. These techniques turn out to be effective in increasing the accuracy of forecasting and in decision-making processes, with growing importance in various applications, as noticed in our paper.

Keywords: Forecasting; Hybrid approaches; Cross-Entropy; Fuzzy-Logic; Attention mechanisms; Accuracy.

1 Introduction

Over the past few decades, there has been a great level of interest in forecasting in several domains, such as the economy, energy, and transportation as indicated by Zhao et al. (2020). Generally, a forecasting method can be classified into traditional methods, modern forecasting methods, and hybrid forecasting methods. Traditional forecasting methods refer to methods that are derived from available working principles, such as conventional time series forecasting methods and econometric methods (Liu et al. (2021)). However, conventional time series forecasting techniques have been extensively used to forecast future values of variables. An alternative approach that has received great interest from researchers is the use of modern forecasting methods. Among these methods, machine learning techniques such as artificial neural networks, support vector machines, and hybrid models receive increasing attention (Kurani, Akshit, et al (2023)). Another popular modern forecasting technique used by researchers includes regression-based algorithms, local models, generalized regression, and time coefficient functions (Hanifi et al. (2020)). Forecasting, as a method of analysis and investigation into future developments and event occurrence, has always generated great interest in many fields, such as technology, engineering, and economics according to Li et al. (2021). Economic theories and engineering knowledge are often used as drivers for making decisions about the future.

Actually, the result of a forecasting method is a fairly accurate evaluation of what will happen in the future regarding certain interest-related topics. In general, forecasting can be defined as an informed guess for future observations, based on known past behaviors. In order to have a superior understanding of processes and system behaviors, it is important to build low-cost and computationally efficient forecasting models. These issues have led Wolpert to argue that a forecasting method that already uses information such as domain and data about the problem outperforms forecasting models that only use data in the model building phase (McAndrew et al. (2021)). Makridakis et al. (2020) states that there is no universally acknowledged method for accurately forecasting future occurrences. Forecasters may implicitly or explicitly place more or less emphasis on certain types of data when making a forecast, and enlisting that data might prove challenging. However, at the base level, at least two approaches can be combined to form a measure that should predict behavior over time better than either source alone. This is what defines the study of hybrid models for forecasting, which seeks to categorize and train disparate forecasting techniques to improve on individual model behavior (Guermoui et al. (2020)).

Moreover, forecasting is the process of making predictions of the future based on past and present data and analysis of trends. According to Seyedan et al. (2020), forecasts are never perfect, but they are necessary to aid the decision-making process in a variety of fields and domains such as finance, logistics, and enviro-political affairs. Forecasts of disasters, such as droughts or earthquakes, allow governments to estimate the level of assistance needed should the event take place. Moreover, stockholders, likewise, benefit from accurate forecasting, as predictions of company performance fluctuate supply and demand for individual stocks. In general, forecasting is mainly used to reduce uncertainty about future events. According to Ghosh et al. (2022) and Huang et al. (2021), regardless of the data's characteristics or the forecaster's goals, a hybrid model emerges as a unified way of dealing with the wide range of characteristics manifested in various subfields and datasets found in the field of forecasting. Various techniques of forecasting can be used to anticipate the future based on the data collected from different sources, as shown by Yin et al. (2021). These include decision theory, stochastic process model, time series-based model, and so on, as described by Chekroud et al. (2021). All of the forecasting techniques have their own space of applicability. The time series analysis provides a trade-off between complexity and performance. The major issue of time series forecasting is underfitting that results in poor forecasting performance. All the forecasting models have their own advantages and limitations.

In fact, forecasting, simulation, and scientific management tools are all equally significant. It saves time and money and helps you plan for future events. By thoroughly investigating corporate productivity, it improves workplace efficiency. It aids in developing sound objectives and policies. Furthermore, it aids in several stages of organizing. It saves both labor resources and operational costs. Forecasting aids in the management of the organization's varied activities and promotes sound decision-making. Also, forecasting facilitates collaboration among an organization's departments, as indicated by Salcedo-Sanz et al. (2020).

The content of the essay is structured as follows. The next section discusses the literature review of past studies. Section 3 outlines the methodology and discusses the evaluation metrics. Section 4 presents case studies with their datasets and presents empirical results; Section 5 provides a discussion of the findings and Section 6 summarizes the essay.

2 Literature Review

Forecasting is a prevalent decision-making instrument in numerous disciplines, and its literature has shown an increasing number of studies as stated by Zellner et al. (2021). Wolnicki

et al. (2021) observed that recent forecasting surveys have mostly focused on new forecasting in the face of rising globalization, global peace, and international competition. The demand for more accurate options in decision-making models has not diminished over time. As a result, researchers have been more interested in this subject, as stated by Phillips-Wren et al. (2020). Generally, two types of forecasting techniques are distinguished: qualitative and quantitative. Qualitative forecasting includes human judgment and survey forecasting. This approach is mostly based on the experience and intuition of individuals. Quantitative forecasting includes time-series forecasting, causal methods, and machine learning. Researchers who have expertise in a particular forecasting method have investigated it. However, if a single technique is used, the results may be overstated. As a result, researchers are increasingly turning to hybrid methodologies. In fact, Hybrid techniques are becoming more popular, with the purpose of integrating the advantages of both methodologies. These hybrid forecasting methods have the potential to produce more accurate predictions while also enhancing and improving visual analytics tools for decision-making or decision-support.

As a matter of fact, data preprocessing, also known as data integration and purification, is the first step in the forecasting process, and prediction is the second as explained by Phan et al. (2022). A hybrid forecasting method takes into account the distinct qualities and challenges of these two processes. Through the use of synergistic combinations of two or more methodologies, this analysis combines dynamics in an effort to comprehend current market patterns. Among the many advantages of the hybrid model are its ability to improve prediction performance and its relative superiority over separate forecasting models in some areas, as stated by Al Mamun et al. (2020). Furthermore, hybrid systems avoid the problems that individual systems might have, such inconsistent or ambiguous results, low reliability, or poor convergence. In conclusion, requiring the outputs of various methodologies to be used concurrently enhances forecast outcomes and eliminates situations where individual procedures may fail.

Furthermore, hybrid approaches to forecasting have recently been gaining immense popularity across industries, including finance, as they can capture complex patterns and enhance the accuracy of predictions. Such models combine many methodologies, like statistical methods and machine learning algorithms, to make use of their individual strengths. Bulut and Hüdaverdi (2022) proposed a hybrid model that combines the ARIMA-GARCH, SVM, and LSTM models in a single framework for predicting the stock market. As the authors say, with

the integration of those methodologies within a single framework, the hybrid model turns out to be better at predicting stock market data from both industrialized and emerging markets compared to using those models one by one. Moreover, Boubaker et al. (2023) introduce a hybrid model, ARFIMA-WLLWNN, combining ARFIMA, wavelet decomposition, and LLWNN neural networks, enhancing forecasting accuracy and outperforming other models using daily Dow Jones returns.

Another work by Patel et al (2023) proposes a hybrid model for stock market prediction using financial news and social media data from Stocktwits. The authors pose this problem as a binary classification task and illustrate that the use of these newer data sources in a hybrid model may add to the predictive performance. Along these lines, Zahouani and Boubaker (2024) assessed the efficiency of diverse combined approaches to forecast Bitcoin prices. The authors find that hybrid models, specifically ANN-LSTM and LSTM-Attention, had generated better forecasting results for the cryptocurrency market in comparison with simpler models. Among the different models considered, the LSTM-Attention model provided the closest forecasts and showed the most confluence with actual market fluctuations. The inclusions of gradient-specific optimization techniques further improved the LSTM-Attention model and enhanced its forecasting capability. These results confirm the efficiency of hybrid methods for the prediction of the erratic nature of markets like cryptocurrency.

In the past few years, the hybrid models have gained ground rapidly in the energy sector as a means to tackle the complexities and uncertainties connected to the energy data floods. Such models are especially applicable for the energy calculations of demand, supply, and prices. Thus, Che et al. (2022) introduced a hybrid methodology of an ARIMA and an LSTM model that was based on the integrated approach of an ARIMA model and an LSTM model. This paper demonstrated the lift in the forecasting accuracy of the load by the hybrid model that the nonlinear and linear aspects of the electricity demand data were handled, respectively. Furthermore, a study by Han et al. (2022) reports a hybrid solution to wind power forecasting that is realized by the juncture of the weather forecast data and ML approaches. The authors, thus, showed an instance that incorporation of those datasets and modeling techniques can better the performance of the hybrid model in the estimations of wind power.

Recently, Hybrid techniques that include entropy and fuzzy logic are increasingly applied in a variety of industries related to forecasts, including financial and energy markets. Another important contribution was made by Chen et al. (2023) in their study which considered the

investigation of a new hybrid model combining Ensemble Empirical Mode Decomposition (EEMD) with fuzzy entropy and Extreme Learning Machine (ELM). According to the authors, with measures of the complexity of hidden patterns in financial time series data, fuzzy entropy is in a position to contribute to higher accuracy in forecasting. Their empirical results suggest that this proposed hybrid model outperforms any other traditional method of forecasting, which is itself important for capturing the unpredictability and complexity of fuzzy logic in financial data. Furthermore, in the energy industry, Mardani et al. (2023) proposed a hybrid model that included intuitionistic fuzzy entropy, Step-wise Weight Assessment Ratio Analysis (SWARA), and Complex Proportional Assessment (COPRAS). The approach is applied to the evaluation of biomass crop types for energy production, highlighting the importance of fuzzy logic when dealing with uncertainty and subjective judgments in any decision-making processes. The paper shows how hybrid methods can be used to make enhanced forecasting and selection in applications related to energy.

All of the existing studies indicate that augmenting entropy with fuzzy logic into a hybrid improves the capability to represent complex, uncertain settings of financial and energy markets. These kinds of approaches excel in improving the accuracy of the methods of forecasting and decision-making processes in the research reviewed and underline their growing significance in a wide range of applications.

3 Methodology

Certain markets are chaotic and complex, making them difficult to predict. However, predicting these markets is crucial for policymaking and evaluating financial and economic system variables. Various techniques in finance and computer science have been used to forecast financial markets. One approach is the principle of entropy, which measures uncertainty and disorder in systems. Entropy holds promise for both short-term and long-term forecasting in finance and energy. Another approach is fuzzy logic, which is often used as an alternative or in conjunction with other forecasting technologies. Hybrid forecasting approaches have been developed, with the financial and energy sectors being top priorities due to their high volatility. Hybrid forecasting models often produce more accurate results than single models. This paper addresses the issue of utilizing entropy and fuzzy logic in new hybrid forecasting approaches to provide better and more accurate results, covering applications related to finance and energy.

3.1 Fuzzy logic and Its Application in Forecasting

Fuzzy logic is one of the greatest applications of handling uncertainty and imprecision in prediction. It allows for an ambiguous concept of modeling and linguistic too, that makes it quite easy to include expert information into forecasting models. In 1965, Zadeh first put forth this idea linking it with human linguistic words. In the 60's, Zadeh, a U.S.-based scientist, introduced the concept for the first time and spoke about its resemblance to human language. Fuzzy logic is a contextual element whose membership in a set is determined not by one or zero binary (0 or 1) but by a degree, which is indicated by a membership function.

The membership function $\mu_A(x)$ of a fuzzy set A in a universe of discourse X is defined as:

$$\mu_A(x) : X \rightarrow [0, 1]$$

Where x is an element in X and $\mu_A(x)$ represents the degree of membership of X in A. The value of μ_A lies between 0 and 1, with 0 indicating no membership and 1 indicating full membership.

3.2 Cross-Entropy Theory and Its Application in Forecasting

Cross-entropy is suitable for evaluating the difference between predicted and actual distributions, which is crucial in forecasting tasks where the goal is to closely align predictions with real values. In fact, in complex and hybrid models, cross-entropy serves as a loss function that effectively adjusts the model's weights, leading to faster convergence and better optimization. Indeed, cross entropy emerges from the field of information theory and quantifies the difference between two probability distributions. This metric is particularly relevant for the asymptotic equipartition property, which serves as the foundation for a variety of data-compression methodologies, as well as the minimization of code length in data-transmission scenarios. Cross entropy is a cost function for two discrete random variables X and Y.

The formal expression of the entropic function maximization in the context of a probability distribution is usually given by the Cross-entropy formula:

$$H(P, Q) = - \sum_{i=1}^n P(x_i) \log_2 Q(x_i) \quad (1)$$

Where:

P is the true distribution,

Q is the predicted distribution,

Cross-entropy quantifies the inefficiency of assuming a Q distribution when the true distribution is P. It is frequently used in machine learning, particularly for classification tasks, to calculate the difference between expected probability and true labels.

3.3 Evaluation Metrics for Forecasting Models

Forecasting models are evaluated based on their accuracy, using various metrics like mean absolute percentage error (MAPE). MAPE is preferred for forecasting and time-series analysis due to its percentage representation. However, it lacks a magnitude presentation depending on the direction of forecast error. Mean squared error (MSE) measures the variation from original values but can be affected by radical deviations and a variety of data points. The root mean squared error (RMSE) addresses the MSE's drawback by taking the square root of the average squared errors. And finally mean absolute error (MAE) as one of the evaluation metrics to determine the prediction accuracy of a forecasting model.

Our study assessed model performance using the four common measures (MAE, MSE, MAPE, RMSE) for machine learning and prediction to estimate predictive efficacy.

$$MAE = \frac{1}{N} \sum_{i=1}^N |y_i - f_i| \quad (2)$$

$$MSE = \frac{1}{N} \sum_{i=1}^N (y_i - f_i)^2 \quad (3)$$

$$MAPE = \frac{100}{N} \sum_{i=1}^N \left| \frac{y_i - f_i}{y_i} \right| \quad (4)$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^N |y(i) - \hat{f}_i|^2}{N}} \quad (5)$$

where N is the quantity of data to be assessed, y_i is the i th true value, and f_i is the i th forecast value.

4 Case studies

4.1 Application in financial markets

Hybrid approaches are being used to forecast the cryptocurrency market, using entropy and fuzzy logic methods. Entropy helps determine the right weights of indicators, while fuzzy logic

predicts cryptocurrency price movement. These methods are useful for risk management and modeling inexact, uncertain, and subjective decision-making processes. This study uses several approaches to analyze and forecast financial trends in cryptocurrency markets using these methods, focusing on different levels at the cryptocurrency market. This approach has shown success in combining information and predicting market trends.

This study analyzes Ethereum price movements from 5 September 2016 to 15 April 2020 using a reliable dataset from Kaggle. The dataset provides a detailed snapshot of Ethereum's price behavior over 4 years. Figure 1 depicts the variations in the price of Ethereum over time, with a daily time scale on the x-axis and the corresponding price values on the y-axis. This visual representation allows us to interpret the data's behavior.



Figure 1: Time Series Plot for Ethereum Price Movements

4.2 Application in Energy markets

Recent years have witnessed a surging research interest in the fuzzy-entropy approach as a benchmark for forecasting directional movements. However, the two methodologies have, to the best of our knowledge, never been integrated beforehand in order to evaluate future energy trends. To this end, this paper seeks to bridge this gap by integrating the information-theoretic concept of entropy and the qualitative approach of fuzzy logic in the energy sector. This integration aims to examine both the possibilities and potentials therein, which will elucidate

and consequently support nascent and future research studies in spheres of application that subtend the field of energy.

This study examines natural gas movements from September 5, 2019, to April 15, 2024, utilizing a credible Kaggle dataset. The dataset gives a detailed snapshot of natural gas price behavior over a five-year period. Figure 2 displays the price changes in natural gas over time, with a daily time scale on the x-axis and the corresponding price values on the y-axis. This visual depiction enables us to interpret the data's behavior.

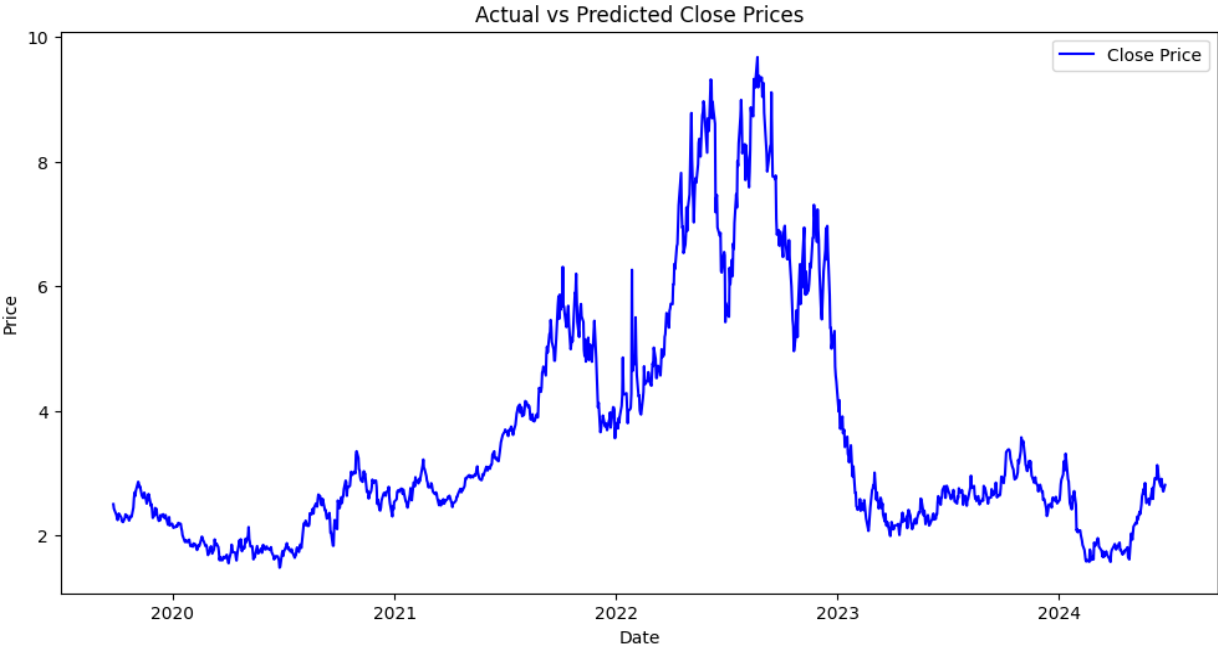


Figure 2: Time Series Plot for Natural Gas Movements

4.3 Empirical Results

This section presents numerical results for the proposed forecasting approaches applied in two different domains: the finance market and the energy market. In each market, we use two models: the first model combines ANN with LSTM, and to enhance its performance, fuzzy logic and cross-entropy are applied separately. While the second model is LSTM, and in order to improve LSTM's performance, we added cross-entropy, then we integrated LSTM with fuzzy logic and finally we combined the LSTM with the attention mechanism. As a matter of fact, entropy allows us to make use of the uncertainty quantification for every prediction, while fuzzy logic helps in handling ambiguity and imprecision in the data.

ANN-LSTM Model for Forecasting Ethereum Prices

This section focuses on using the hybrid ANN-LSTM model. The hybrid technique aims to forecast Ethereum's daily price. Table 1 shows the projected and actual values for the most recent 10 observations.

Table 1: Actual and Predicted Daily Ethereum Prices Using the ANN-LSTM Model

Date	Actual price	Predicted price
04-07-2019	283.86	296.701111
05-07-2019	288.05	296.630188
06-07-2019	287.87	293.000977
07-07-2019	306.58	295.211060
08-07-2019	313.63	300.393005
09-07-2019	308.28	312.253082
10-07-2019	288.73	316.781006
11-07-2019	267.72	305.776245
12-07-2019	275.59	284.670258
13-07-2019	269.52	277.554260

Figure 3 compares given and actual values for the ANN LSTM model, offering a clear evaluation of its performance. To assess the model's efficacy, four performance indicators were used: mean square error (MSE), mean absolute error (MAE), mean absolute percentage error (MAPE) and the root mean squared error (RMSE) and Table 2 shows the MSE, MAE, MAPE, RMSE values, indicating the hybrid model's predictive accuracy. Figure 4 demonstrates how the model's performance has improved over time. As the number of epochs increases, the loss decreases, showing that the model is improving its predictions. The model's total accuracy is 97%.

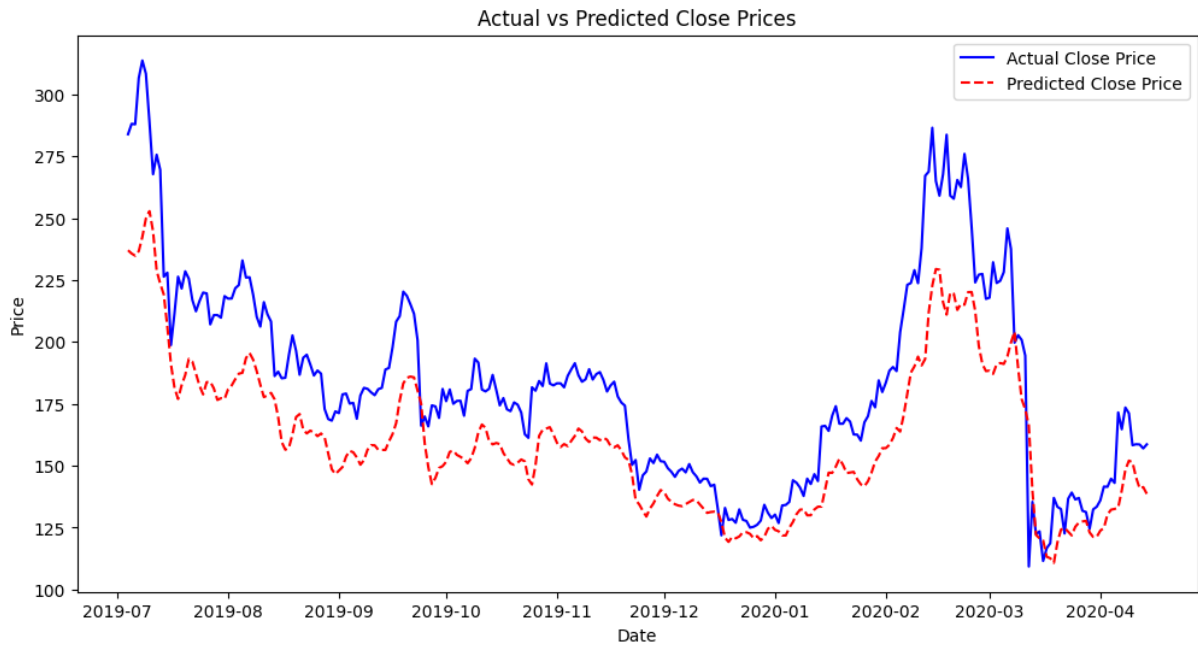


Figure 3: Predicted and Actual Ethereum Prices Using ANN-LSTM

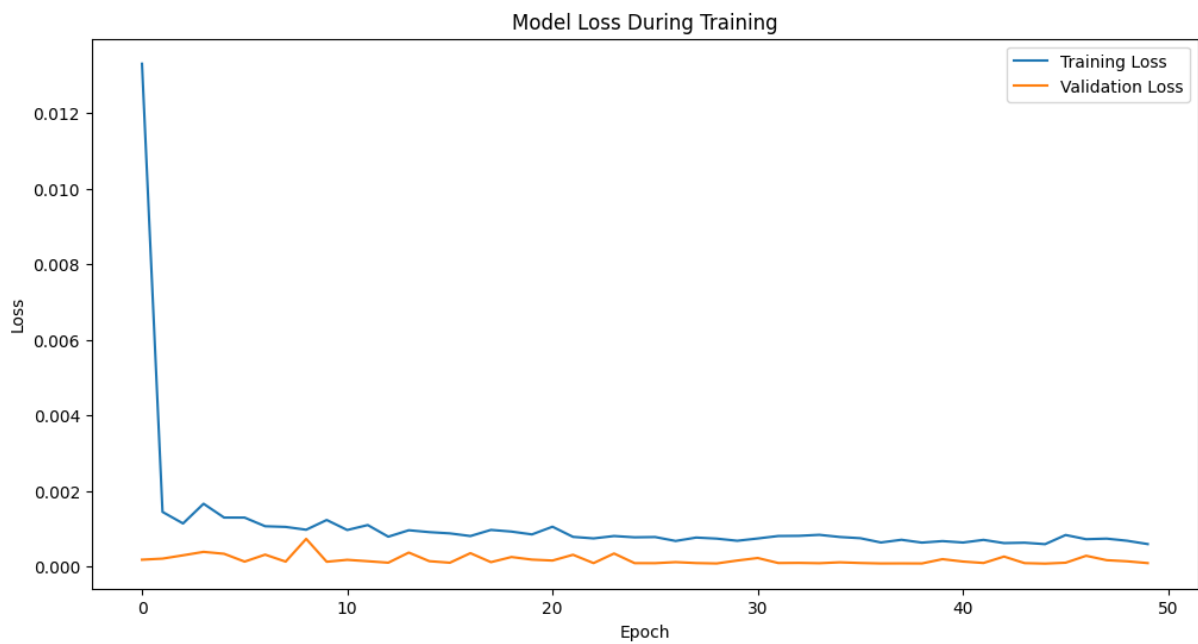


Figure 4: Training and Validation Loss of ANN-LSTM Model

Table 2: Evaluation Metrics of the ANN-LSTM Model

MAE	MSE	RMSE	MAPE	Accuracy
23.8088	754.9796	27.4768	12.2684%	97%

ANN-LSTM Model for Forecasting Natural Gas Prices

This section focuses on applying the hybrid ANN-LSTM model. The hybrid technique seeks to forecast natural gas daily prices. Table 3 displays the projected and actual values for the most recent ten observations.

Table 3: Actual and Predicted Daily Natural Gas Prices Using the ANN-LSTM Model

Date	Actual price	Predicted price
25-09-2025	2.502	2.576369
26-09-2025	2.428	2.549544
27-09-2025	2.404	2.558221
30-09-2025	2.330	2.465924
01-09-2025	2.283	2.414893
02-09-2025	2.247	2.350034
03-09-2025	2.329	2.295851
04-09-2025	2.352	2.295851
07-09-2025	2.303	2.325840
08-09-2025	2.288	2.330171

Figure 5 contrasts the supplied and actual values of the ANN LSTM model, providing a clear evaluation of its performance. As with the first dataset, four performance measures were employed to evaluate the model's efficacy: mean square error (MSE), mean absolute error (MAE), mean absolute percentage error (MAPE), and root mean squared error (RMSE). Table 4 displays the MSE, MAE, MAPE, and RMSE values, which represent the hybrid model's predictive accuracy. Figure 6 shows how the model's performance has increased with time. As the number of epochs increases, the loss reduces, indicating that the model's predictions are improving. The model's overall accuracy is 99%.

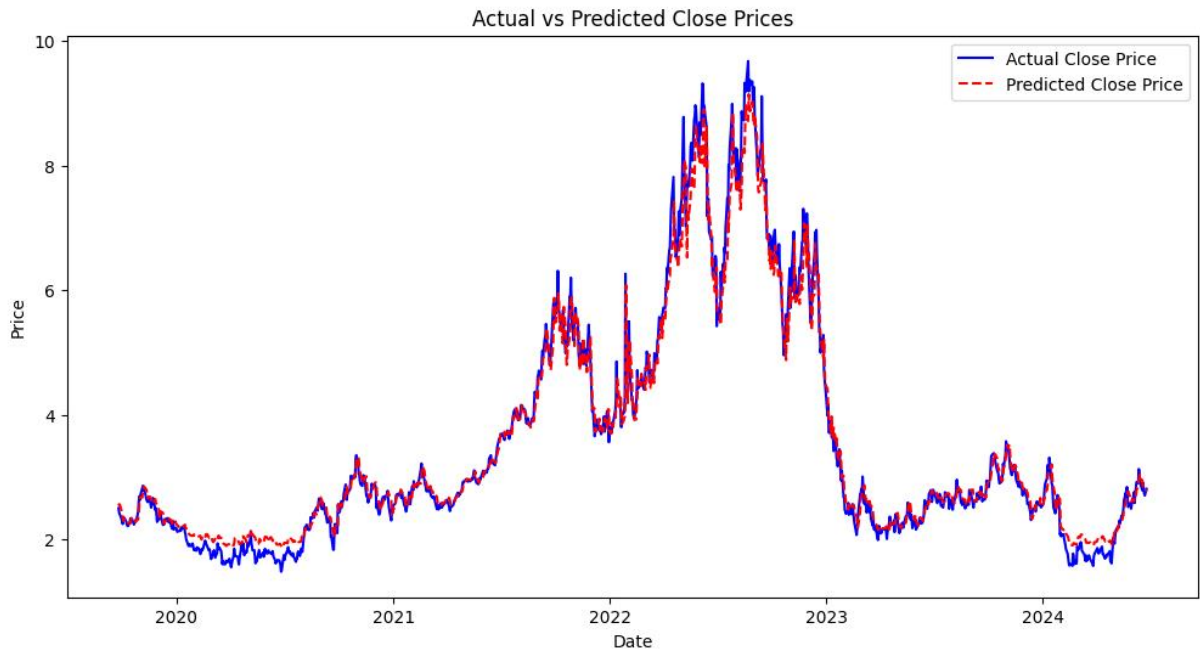


Figure 5: Predicted and Actual Natural Gas Prices Using ANN-LSTM



Figure 6: Training and Validation Loss of ANN-LSTM model

Table 4: Evaluation metrics of ANN-LSTM model

MAE	MSE	RMSE	MAPE	Accuracy
0.1808	0.0717	0.2678	5.4972%	99%

ANN-LSTM with fuzzy logic Model for Forecasting Ethereum Prices

In this part, we incorporated fuzzy logic into our hybrid ANN-LSTM model to enhance its performance. This addition aims to enrich the model's ability to handle uncertainty and imprecision in the data, leading to more robust and accurate predictions.

Table 5: Actual and Predicted Daily Ethereum Prices Using the ANN-LSTM Model with Fuzzy Logic

Date	Actual price	Predicted price
04-07-2019	283.86	292.880768
05-07-2019	288.05	295.386322
06-07-2019	287.87	292.381714
07-07-2019	306.58	293.014557
08-07-2019	313.63	297.095062
09-07-2019	308.28	306.862427
10-07-2019	288.73	313.880188
11-07-2019	267.72	307.397308
12-07-2019	275.59	289.461914
13-07-2019	269.52	278.177673

Figure 7 illustrates the variance between the forecasted and actual values for the hybrid ANN-LSTM model with fuzzy logic, demonstrating very close predictions compared to the real values. According to Table 6, the ANN-LSTM model with fuzzy logic outperforms the standard ANN-LSTM, achieving an accuracy of 98%.

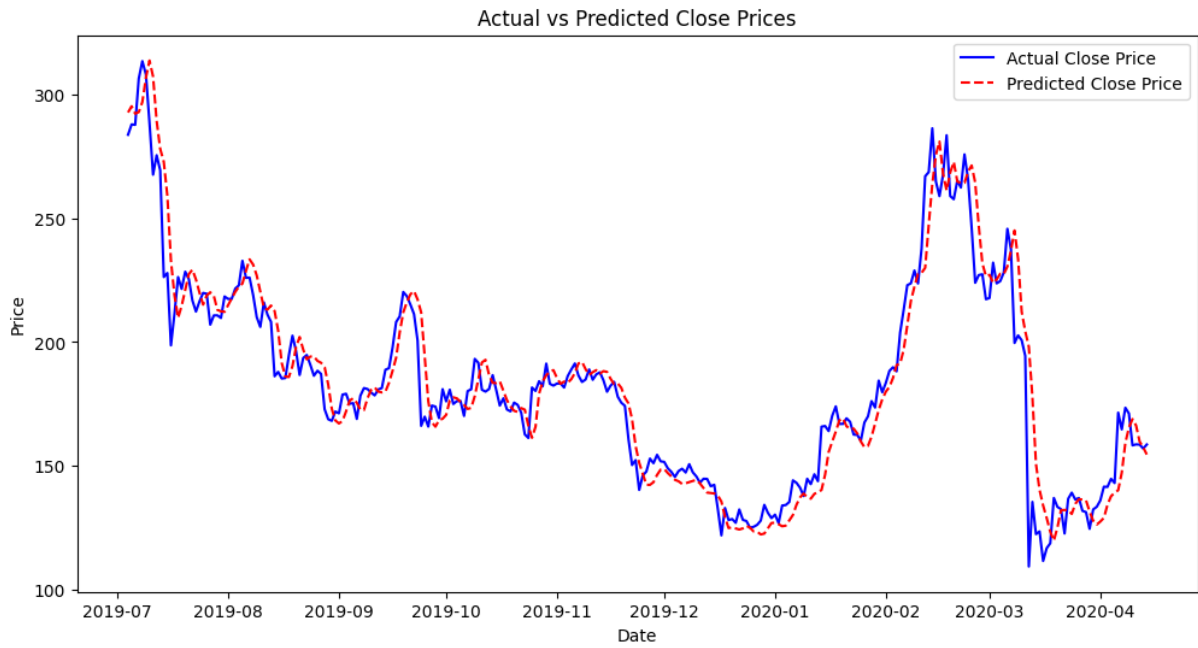


Figure 7: Predicted and Actual Ethereum Prices Using ANN-LSTM Model with Fuzzy-Logic

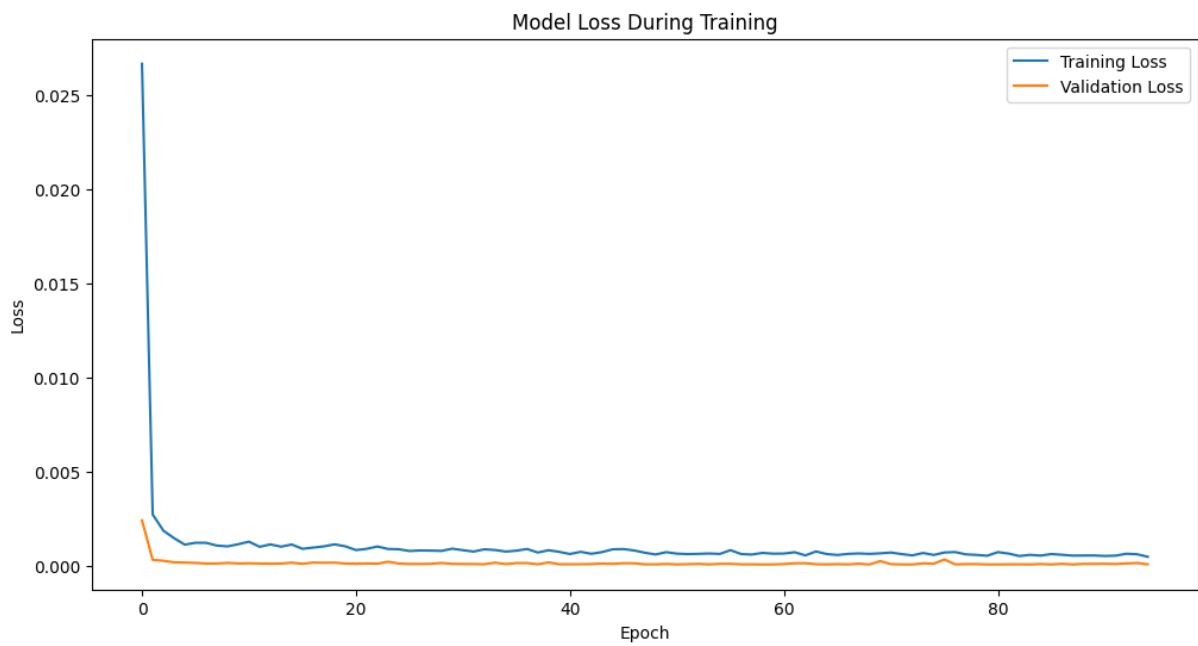


Figure 8: Training and Validation Loss of ANN-LSTM Model with Fuzzy Logic

Table 6: Evaluation metrics of ANN-LSTM model with Fuzzy Logic

MAE	MSE	RMSE	MAPE	Accuracy
8.6202	167.3954	12.9381	4.8461%	98%

ANN-LSTM with Fuzzy-Logic for Forecasting Natural Gas Prices

In this section, we incorporated fuzzy logic into our hybrid ANN-LSTM model to achieve better estimations. Table 7 presents the predicted and actual values of natural gas prices over time. Figure 9 illustrates the plot of these values. The model's performance accuracy is 99%.

Table 7: Actual and Predicted Daily Natural Gas Prices Using the ANN-LSTM Model with Fuzzy-Logic

Date	Actual price	Predicted price
25-09-2025	2.502	2.500881
26-09-2025	2.428	2.377293
27-09-2025	2.404	2.423836
30-09-2025	2.330	2.397307
01-09-2025	2.283	2.357648
02-09-2025	2.247	2.318281
03-09-2025	2.329	2.248549
04-09-2025	2.352	2.289935
07-09-2025	2.303	2.200503
08-09-2025	2.288	2.253746

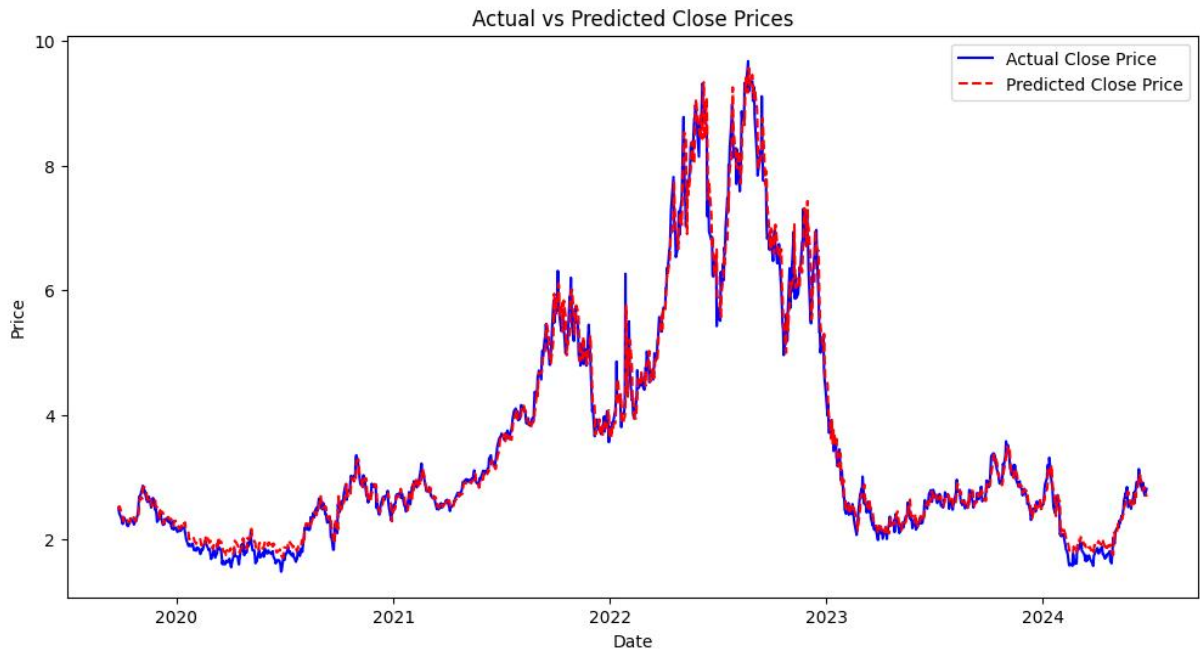


Figure 9: Predicted and Actual Natural Gas Prices Using ANN-LSTM Model with Fuzzy-Logic

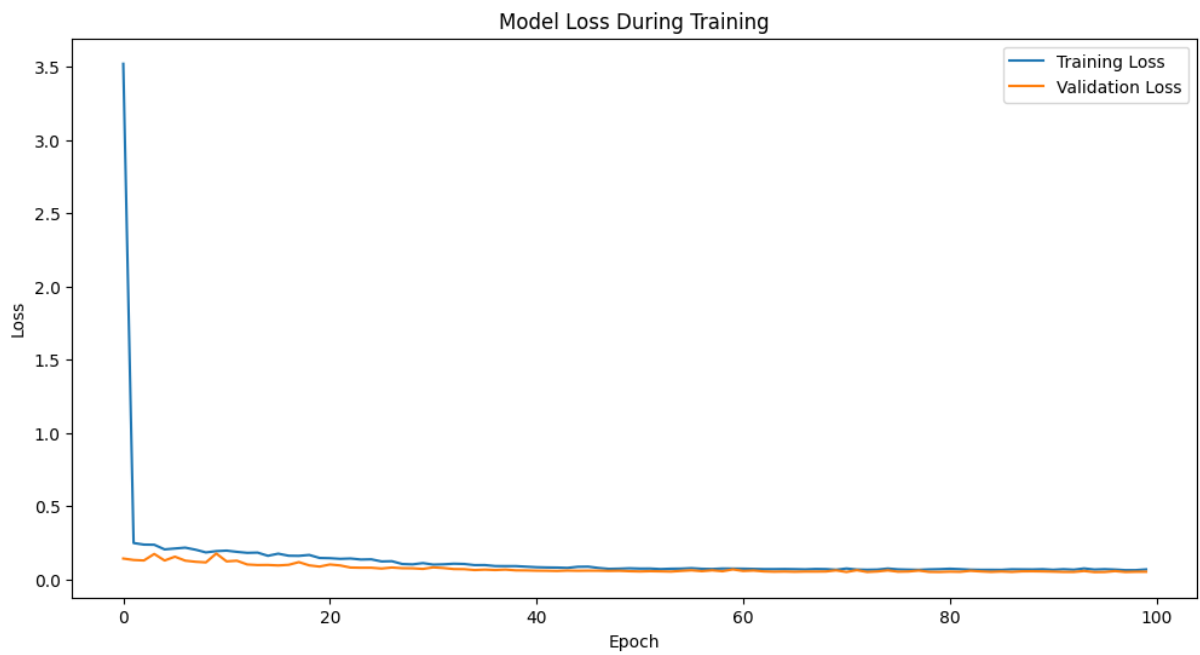


Figure 10: Training and Validation Loss of ANN-LSTM model with fuzzy logic

Table 8: Evaluation metrics of ANN-LSTM model with Fuzzy Logic

MAE	MSE	RMSE	MAPE	Accuracy
0.1537	0.0530	0.2303	4.5420%	99%

ANN-LSTM with Cross-entropy for Forecasting Ethereum Prices

In this part, we incorporated cross-entropy into our hybrid ANN-LSTM model to enhance its performance. This addition aims to improve the model's ability to distinguish between the predicted and actual values, thereby optimizing the overall forecasting accuracy

Table 9: Actual and Predicted Daily Ethereum Prices Using the ANN-LSTM Model with Cross-Entropy

Date	Actual price	Predicted price
04-07-2019	283.86	289.129395
05-07-2019	288.05	291.071075
06-07-2019	287.87	287.724152
07-07-2019	306.58	288.380585
08-07-2019	313.63	291.223114
09-07-2019	308.28	301.652100
10-07-2019	288.73	308.632385
11-07-2019	267.72	301.964355
12-07-2019	275.59	284.837433
13-07-2019	269.52	275.151886

Figure 11 illustrates the variance between forecasted and actual for the hybrid ANN-LSTM model with cross-entropy. Table 10 lists the performance metrics such as MSE, MAE, MAPE, and RMSE values, which are considerably lower than those of the hybrid model. The model Total accuracy is 99% which justifies the necessities of incorporating entropy for enhancing precision in forecasting.

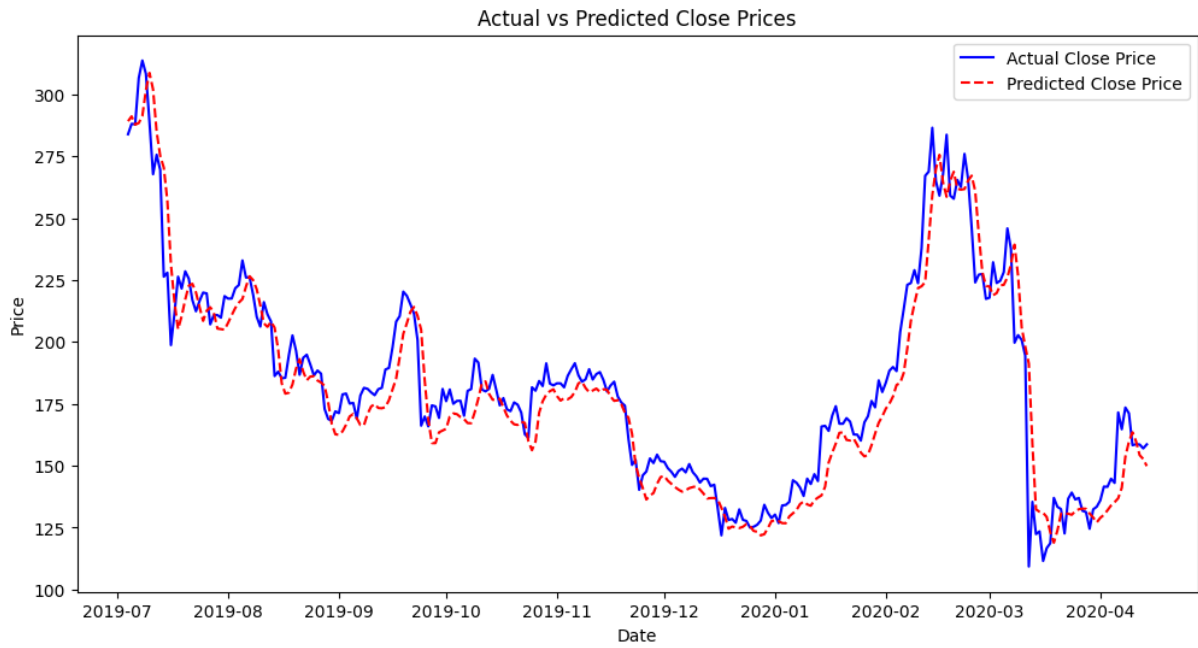


Figure 11: Predicted and Actual Ethereum Prices Using ANN-LSTM Model with Cross-Entropy

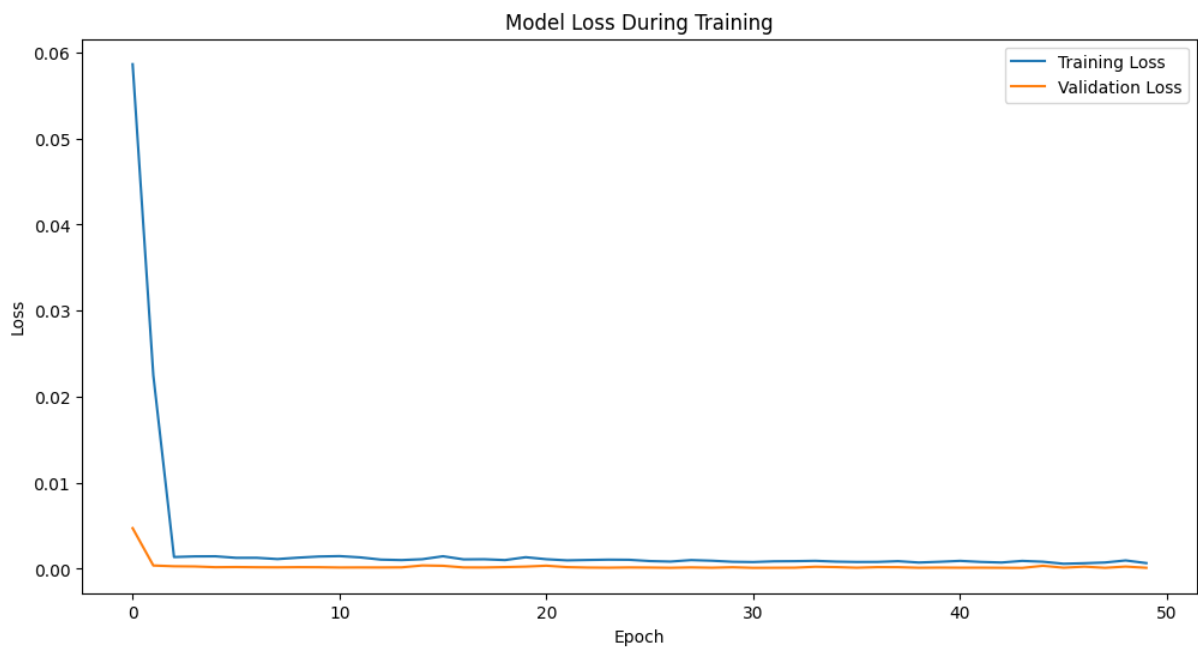


Figure 12: Training and Validation Loss of ANN-LSTM Model with Cross-Entropy

Table 10: Evaluation metrics of ANN-LSTM model with Cross-Entropy

MAE	MSE	RMSE	MAPE	Accuracy
9.7276	175.4941	13.2474	5.3993%	99%

ANN-LSTM with Cross-entropy for Forecasting Natural Gas Prices

In this section, we added cross-entropy to our hybrid ANN-LSTM model to improve its performance. This modification intends to increase the model's capacity to discriminate between expected and real values, therefore maximizing the overall forecasting accuracy.

Table 11: Actual and Predicted Daily Natural Gas Prices Using the ANN-LSTM Model with Cross-Entropy

Date	Actual price	Predicted price
25-09-2025	2.502	2.502001
26-09-2025	2.428	2.477293
27-09-2025	2.404	2.433236
30-09-2025	2.330	2.407307
01-09-2025	2.283	2.357648
02-09-2025	2.247	2.318771
03-09-2025	2.329	2.248549
04-09-2025	2.352	2.220135
07-09-2025	2.303	2.200206
08-09-2025	2.288	2.203764

Figure 13 depicts the variation between predicted and actual values for the hybrid ANN-LSTM model with cross-entropy. Table 12 shows the performance metrics MSE, MAE, MAPE, and RMSE values.

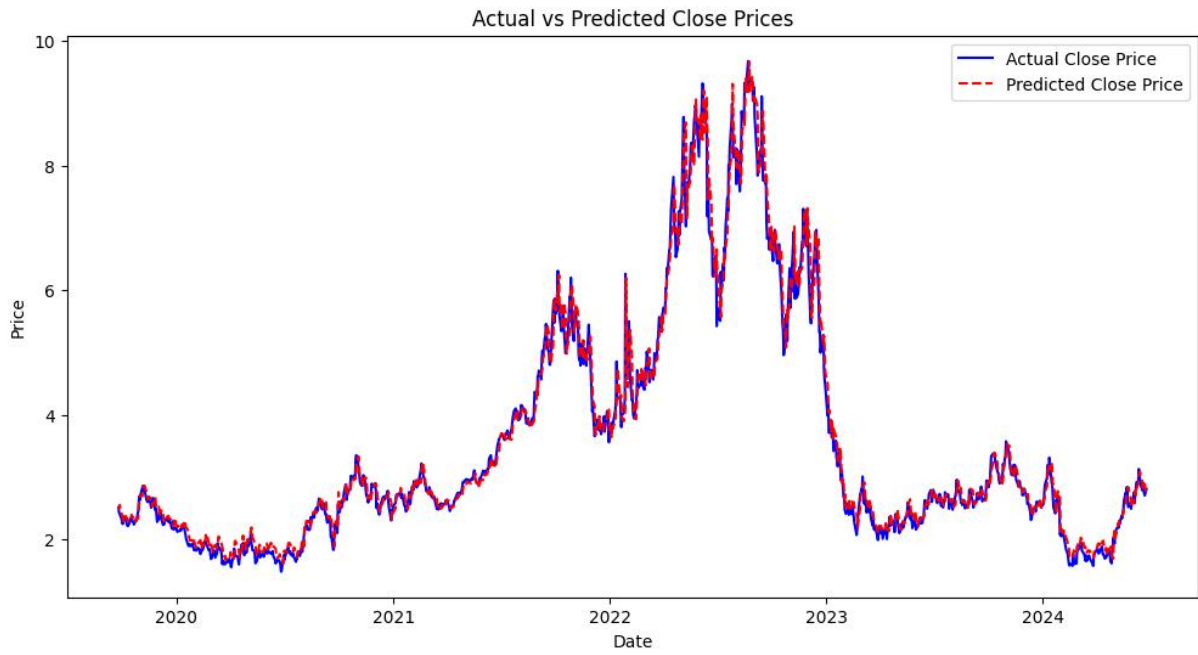


Figure 13: Predicted and Actual Natural Gas Prices Using ANN-LSTM model with Cross-Entropy

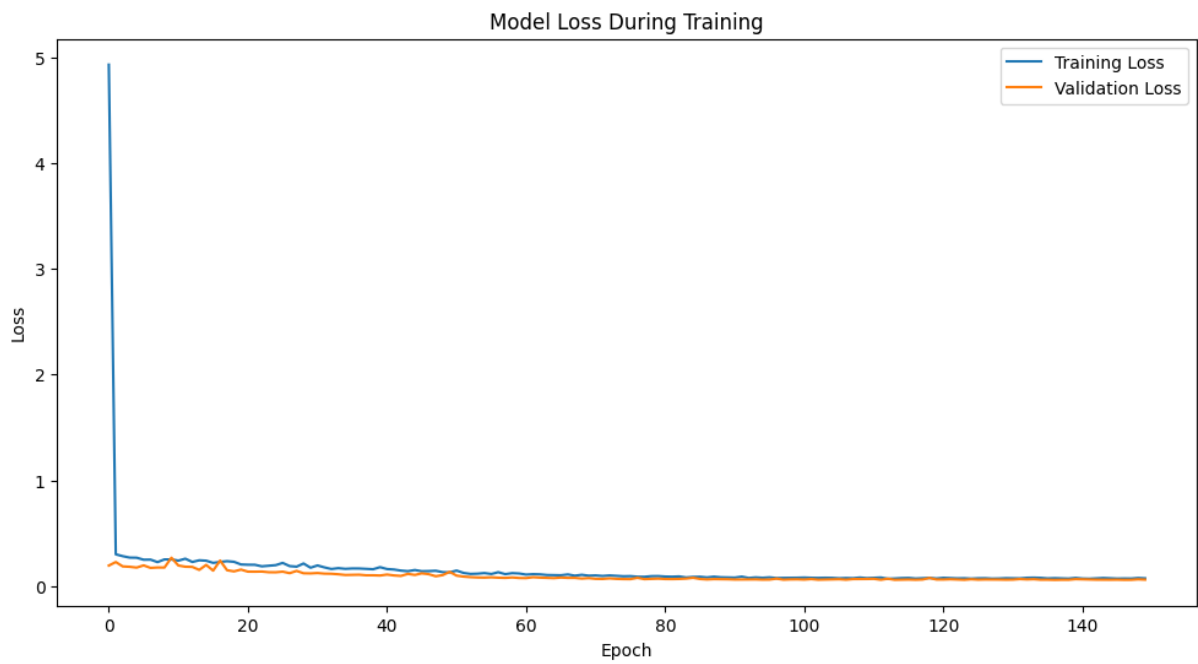


Figure 14: Training and Validation Loss of ANN-LSTM model with Cross-entropy

Table 12: Evaluation metrics of ANN-LSTM model with Cross-entropy

MAE	MSE	RMSE	MAPE	Accuracy
0.1598	0.0606	0.2462	4.5495%	99%

LSTM with Fuzzy Logic for Forecasting Ethereum Prices

In this part, we extended the LSTM model by incorporating fuzzy logic to enhance its performance. This kind of incorporation would empower the model to handle naturally occurring uncertainties and vagueness in real data in much better ways, thereby resulting in an improved decision-making process. It can learn better and finer details from the data for more accurate and reliable prediction.

Table 13: Actual and predicted Ethereum's daily price using LSTM model with Fuzzy-Logic

Date	Actual price	Predicted price
04-07-2019	283.86	303.031494
05-07-2019	288.05	304.442291
06-07-2019	287.87	300.568390
07-07-2019	306.58	301.664581
08-07-2019	313.63	304.974121
09-07-2019	308.28	315.239349
10-07-2019	288.73	321.184418
11-07-2019	267.72	313.829193
12-07-2019	275.59	295.627625
13-07-2019	269.52	286.274017

Figure 15 depicts the difference in expected and actual values for the hybrid LSTM model with fuzzy logic. The LSTM model with fuzzy logic achieved an accuracy of 96%, as shown in Table 14.

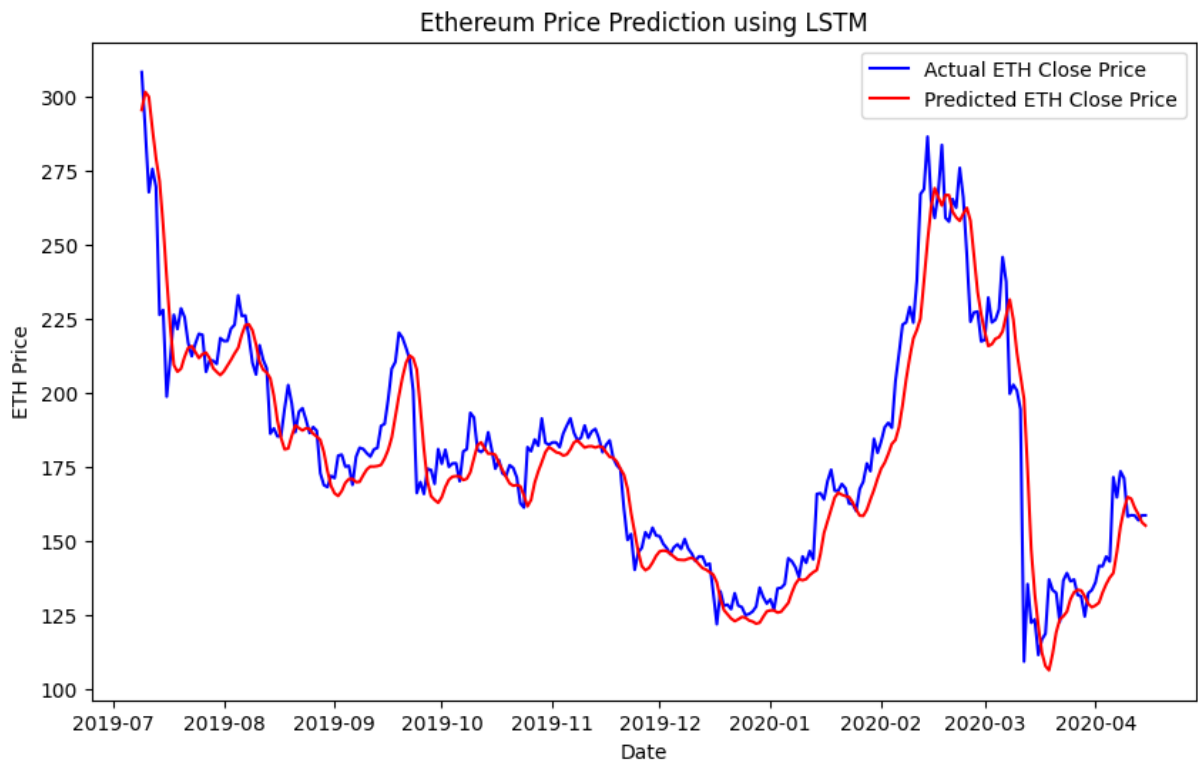


Figure 15: Predicted and Actual Ethereum Prices Using LSTM Model with Fuzzy-Logic

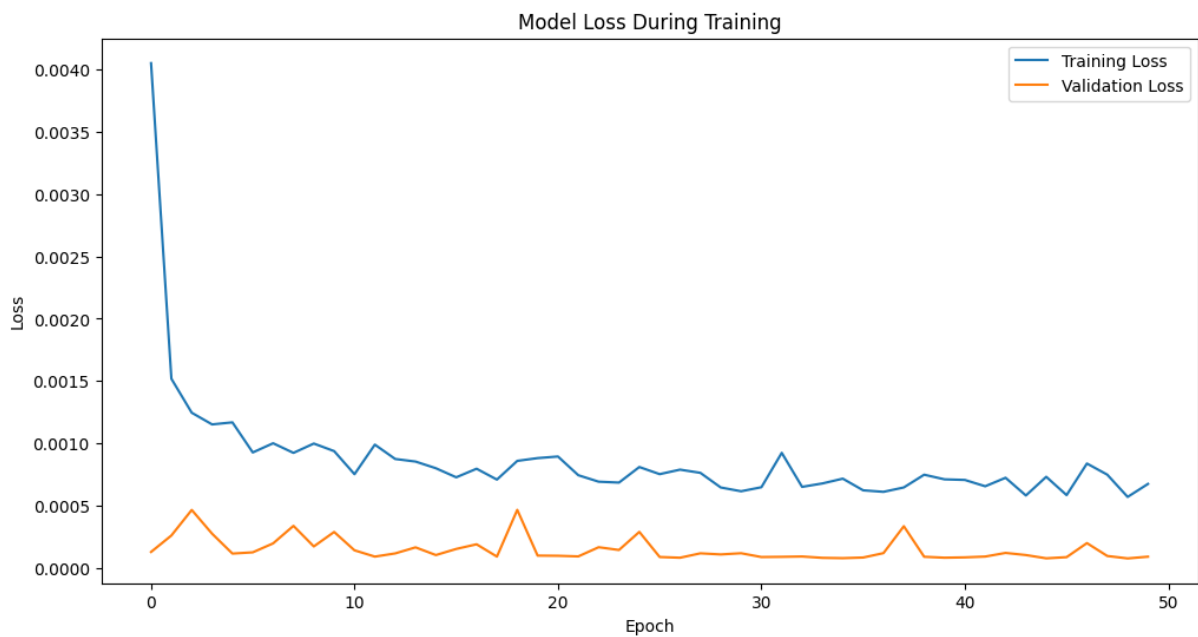


Figure 16: Training and Validation Loss of LSTM model with Fuzzy Logic

Table 14: Evaluation metrics of LSTM model with Fuzzy Logic

MAE	MSE	RMSE	MAPE	Accuracy
488.8095	240424.6334	490.3311	283.9285%	96%

LSTM with fuzzy logic for Forecasting Natural Gas Prices

In this final section, we incorporated fuzzy logic into the LSTM model to improve the results. Table 15 presents our model's estimations, while Figure 17 shows a comparison between the actual and estimated values, providing a clear visualization. Finally, all the evaluation metrics are summarized in Table 16. The model achieved an accuracy of 95%.

Table 15 :Actual and predicted Natural Gas's daily price using LSTM model with Fuzzy-Logic

Date	Actual price	Predicted price
25-09-2025	2.502	2.577638
26-09-2025	2.428	2.423984
27-09-2025	2.404	2.369872
30-09-2025	2.330	2.498722
01-09-2025	2.283	2.311587
02-09-2025	2.247	2.169833
03-09-2025	2.329	2.266932
04-09-2025	2.352	2.369846
07-09-2025	2.303	2.225944
08-09-2025	2.288	2.300871

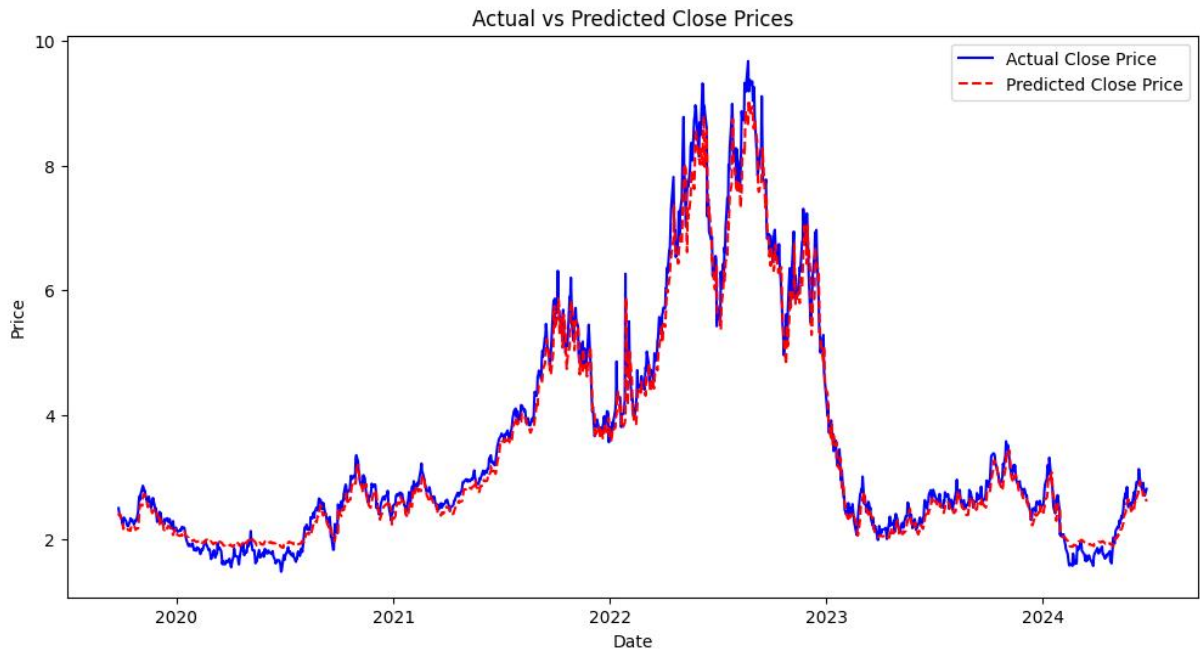


Figure 17: Predicted and Actual Natural Gas Prices Using LSTM Model with Fuzzy-Logic

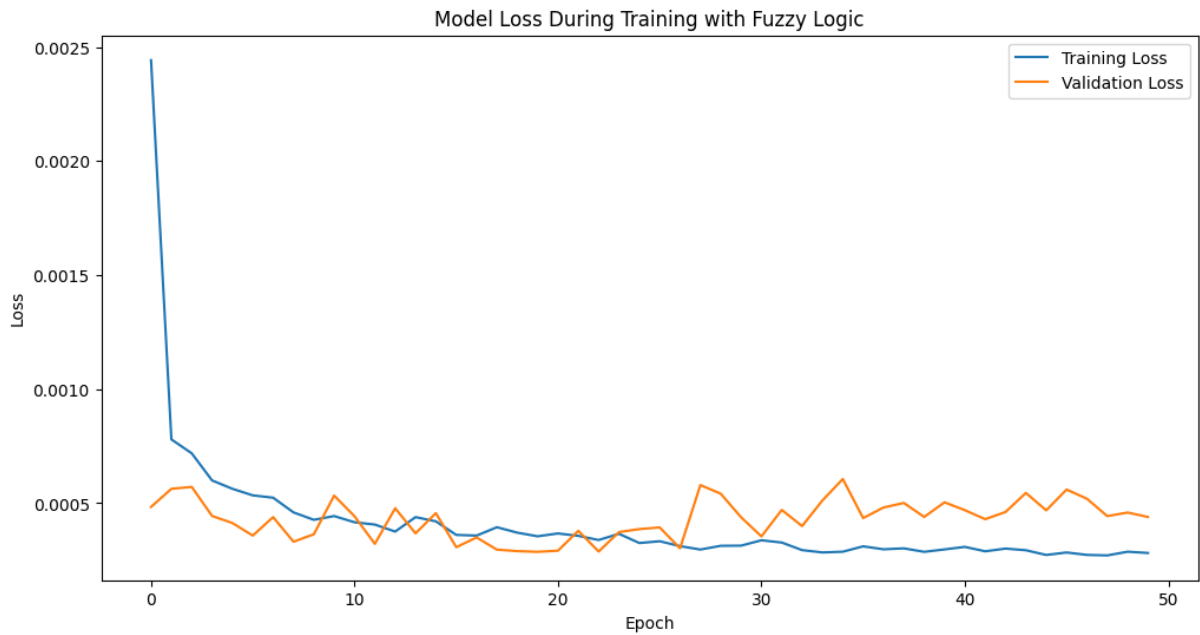


Figure 18: Training and Validation Loss of LSTM model with Fuzzy Logic

Table 16: Evaluation metrics of LSTM model with Fuzzy Logic

MAE	MSE	RMSE	MAPE	Accuracy
139.1647	21551.5044	146.8043	4277.1636%	95%

LSTM with Cross-entropy for Forecasting Ethereum Prices

In this phase, we added Cross-entropy to our hybrid LSTM model to improve its performance. This modification seeks to increase the model's capacity to discriminate between projected and real values, hence increasing overall forecasting accuracy.

Table 17: Actual and predicted Ethereum's daily price using LSTM model with Cross-Entropy

Date	Actual price	Predicted price
04-07-2019	283.86	267.334778
05-07-2019	288.05	265.363220
06-07-2019	287.87	262.997498
07-07-2019	306.58	262.893860
08-07-2019	313.63	264.156097
09-07-2019	308.28	266.737305
10-07-2019	288.73	270.964020
11-07-2019	267.72	274.046906
12-07-2019	275.59	271.412384
13-07-2019	269.52	265.341156

Figure 19 depicts the variation between predicted and actual values for the hybrid LSTM model with cross-entropy. Table 18 shows the performance metrics MSE, MAE, MAPE, and RMSE values. The model Overall accuracy is 97%.

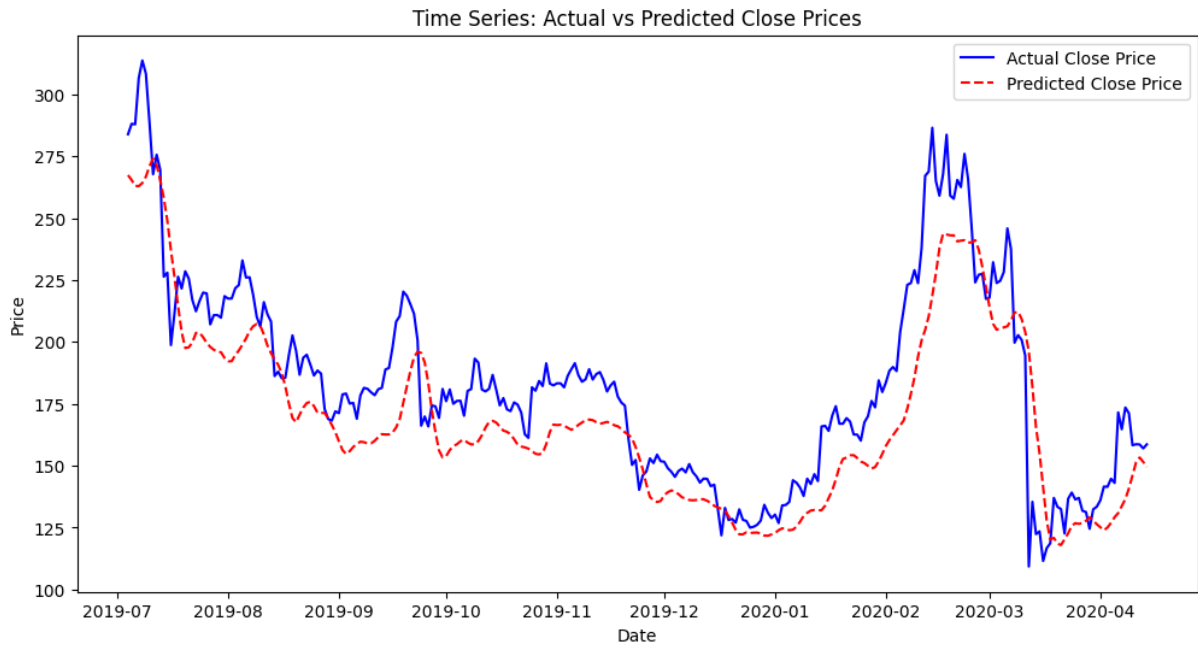


Figure 19: Predicted and Actual Ethereum Prices Using LSTM Model with Cross-Entropy

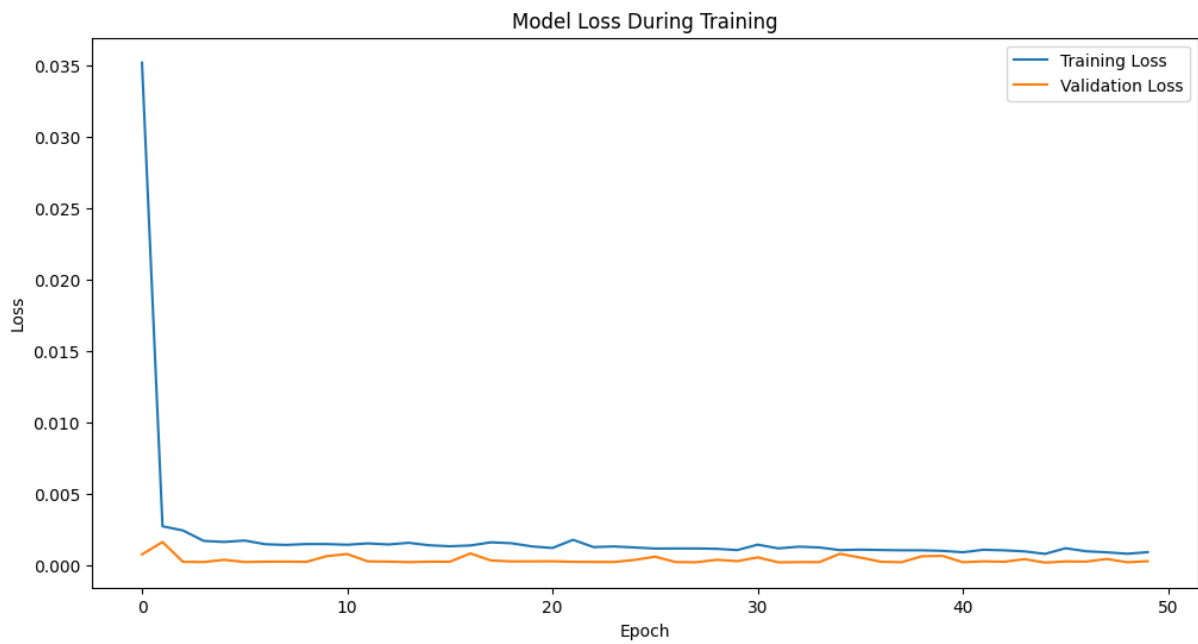


Figure 20: Training and Validation Loss of LSTM with Cross-entropy

Table 18: Evaluation metrics of LSTM model with Cross-Entropy

MAE	MSE	RMSE	MAPE	Accuracy
18.1591	483.5446	21.9896	9.78781%	97%

LSTM with Cross-entropy for Forecasting Natural Gas Prices

During this phase, we improved the performance of our hybrid LSTM model by adding Cross-entropy. Cross-entropy helps train the model to enhance classification accuracy, making it suitable for tasks such as time series classification or forecasting with discrete outcomes. Table 19 presents actual and predicted prices using LSTM model with Cross-Entropy.

Figure 21 shows the difference in predicted and real values for the hybrid LSTM model with cross-entropy. Table 20 displays the performance metrics MSE, MAE, MAPE, and RMSE values. The model Overall, accuracy is 99%.

Table 19: Actual and predicted Natural Gas's daily price using LSTM model with Cross - Entropy

Date	Actual price	Predicted price
25-09-2025	2.502	2.510111
26-09-2025	2.428	2.467292
27-09-2025	2.404	2.433836
30-09-2025	2.330	2.397361
01-09-2025	2.283	2.357648
02-09-2025	2.247	2.318281
03-09-2025	2.329	2.248549
04-09-2025	2.352	2.299935
07-09-2025	2.303	2.280503
08-09-2025	2.288	2.263746

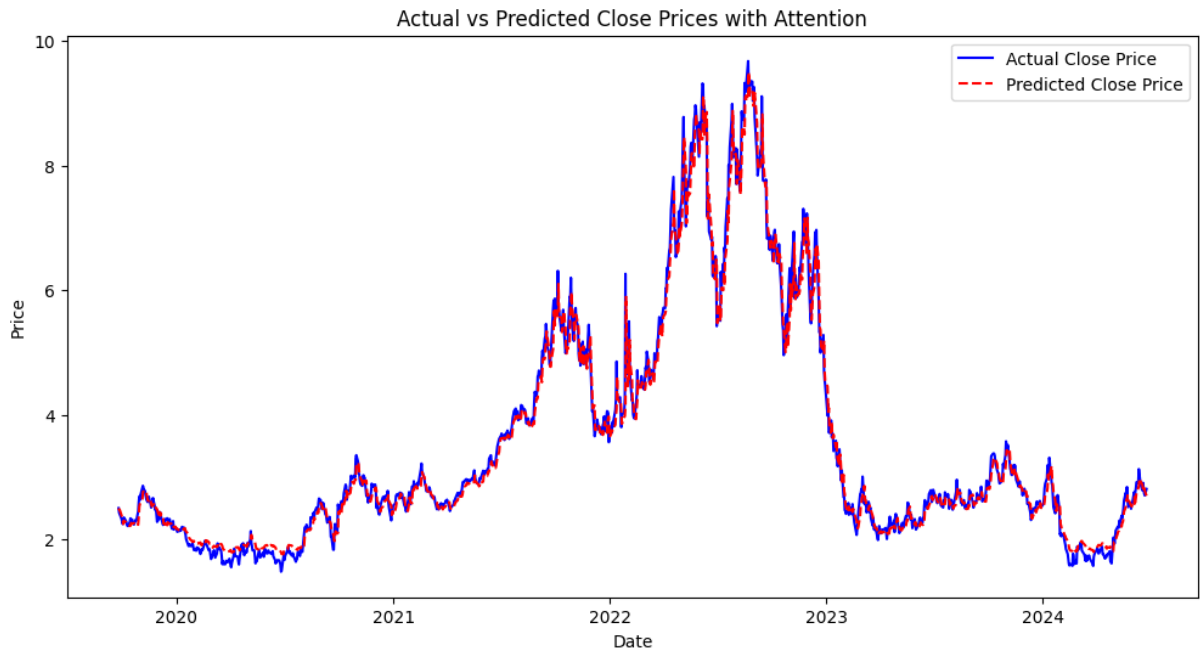


Figure 21: Predicted and Actual Natural Gas Prices Using LSTM Model with Cross-Entropy

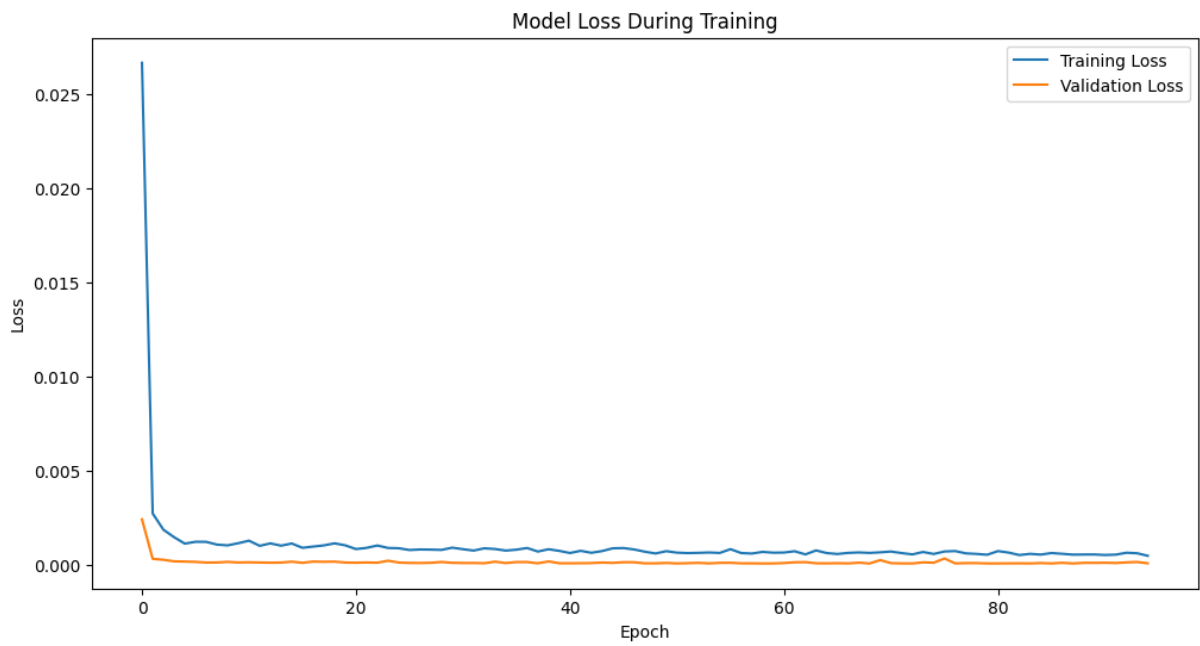


Figure 22: Training and Validation Loss of LSTM model with Cross-Entropy

Table 20: Evaluation metrics of LSTM model with Cross-Entropy

MAE	MSE	RMSE	MAPE	Accuracy
1.6359	4.4070	2.0992	28.7822 %	99%

LSTM-ATTENTION for Forecasting Ethereum Prices

The following part focuses on improving LSTM model by integrating attention mechanisms with Cross-entropy, and separately, by combining LSTM with fuzzy logic. Table 21 displays the projected values using LSTM model and the actual values for the most recent 10 observations of Ethereum's daily price, based on the previously mentioned dataset.

Table 21: Actual and predicted Ethereum's daily price using LSTM-Attention model

Date	Actual price	Predicted price
04-07-2019	283.86	292.482483
05-07-2019	288.05	294.009583
06-07-2019	287.87	290.897797
07-07-2019	306.58	291.200897
08-07-2019	313.63	299.889099
09-07-2019	308.28	304.715485
10-07-2019	288.73	300.569397
11-07-2019	267.72	377.723877
12-07-2019	275.59	281.784332
13-07-2019	269.52	270.354004

Figure 23 shows expected and actual values using the LSTM model, providing a clear evaluation of its performance. As previously stated, four performance measures were utilized to assess the model's efficacy and presented in Table 22. Figure 24 shows how the model's performance has increased with time. As the number of epochs increases, the loss reduces, indicating that the model's predictions are improving. The model's overall accuracy is 98%.

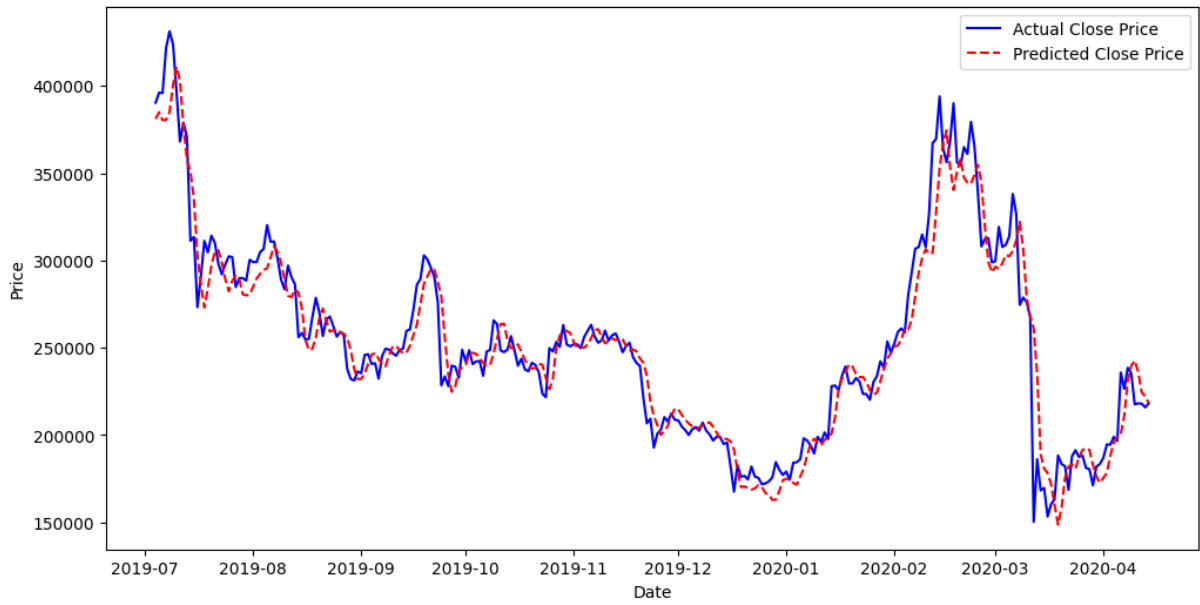


Figure 23: Predicted and Actual Ethereum Prices Using LSTM-ATTENTION Model

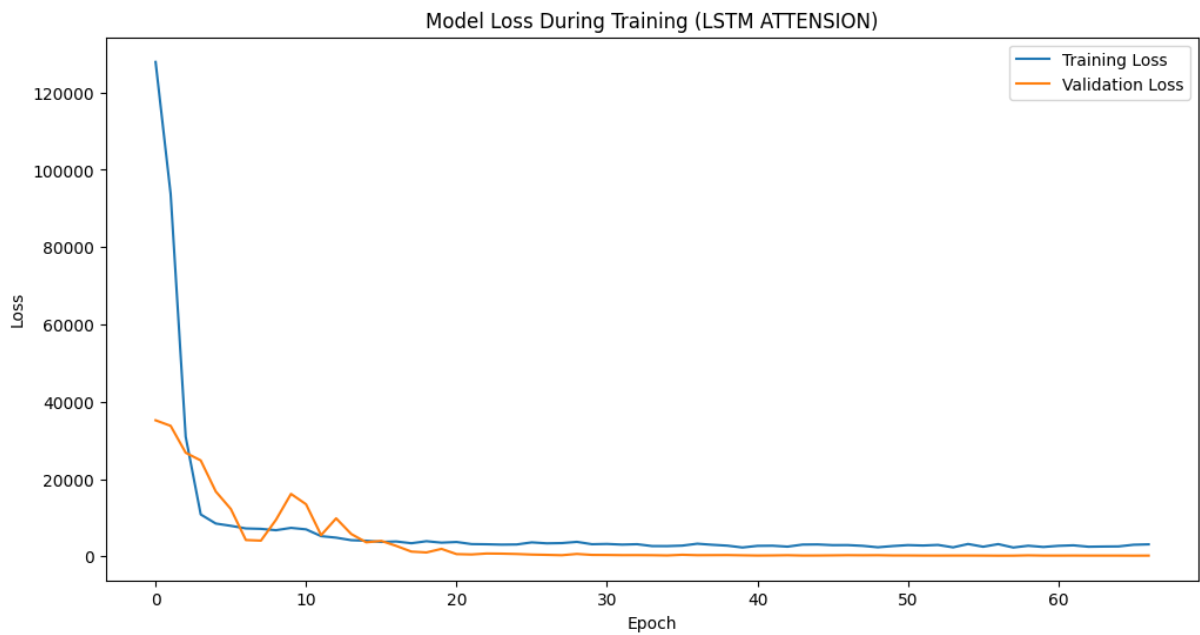


Figure 24: Training and Validation Loss of LSTM-ATTENTION Model.

Table 22: Evaluation metrics of LSTM-ATTENTION model

MAE	MSE	RMSE	MAPE	Accuracy
7.8652	150.2508	12.2576	4.3401%	99%

LSTM-ATTENTION for Forecasting Natural Gas Prices

The following section focuses on strengthening the LSTM model by merging attention processes with Cross-entropy and, separately, combining LSTM and fuzzy logic.

Table 23 shows the projected values using the LSTM model as well as the actual values for the most recent 10 observations of natural gas's daily price, which are based on the second previously stated dataset.

Table 23: Actual and predicted Natural Gas's daily price using LSTM-ATTENTION model

Date	Actual price	Predicted price
25-09-2025	2.502	2.604603
26-09-2025	2.428	2.587835
27-09-2025	2.404	2.589166
30-09-2025	2.330	2.311508
01-09-2025	2.283	2.276868
02-09-2025	2.247	2.441202
03-09-2025	2.329	2.350814
04-09-2025	2.352	2.442590
07-09-2025	2.303	2.427139
08-09-2025	2.288	2.404267

Figure 25 compares expected and actual values for the LSTM model, offering an obvious assessment of its effectiveness. As previously indicated, four performance metrics were applied to assess the model's efficacy, which are shown in Table 24. The model achieves a total accuracy of 99%.

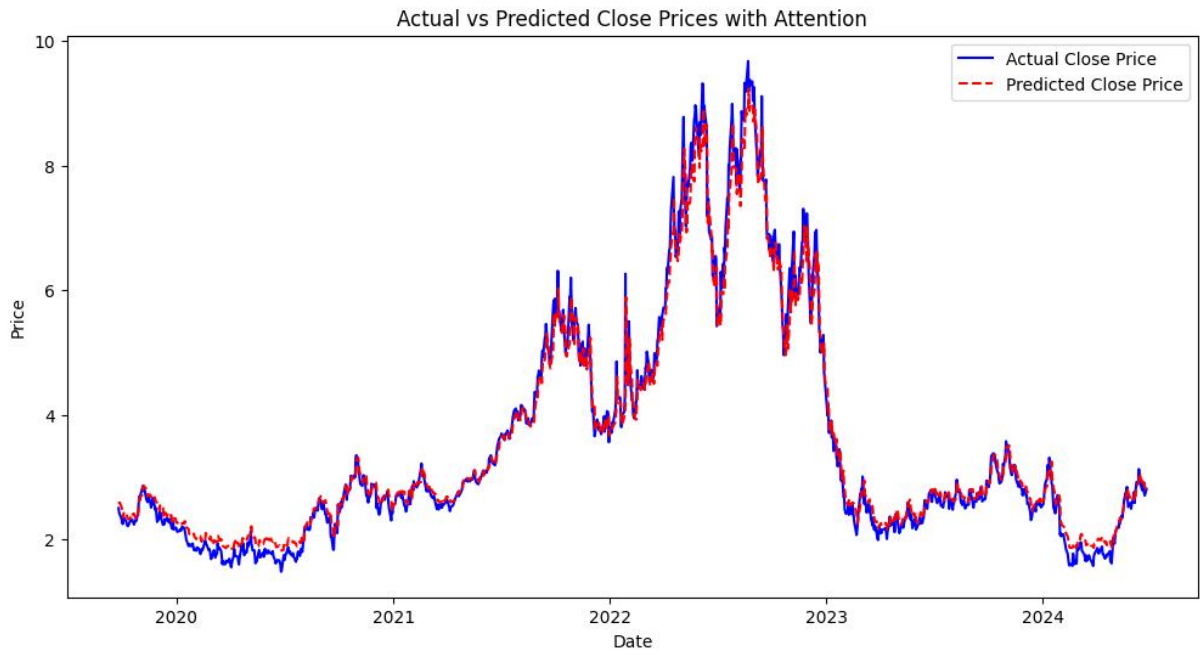


Figure 25 Predicted and Actual Natural Gas Prices Using LSTM-ATTENTION Model

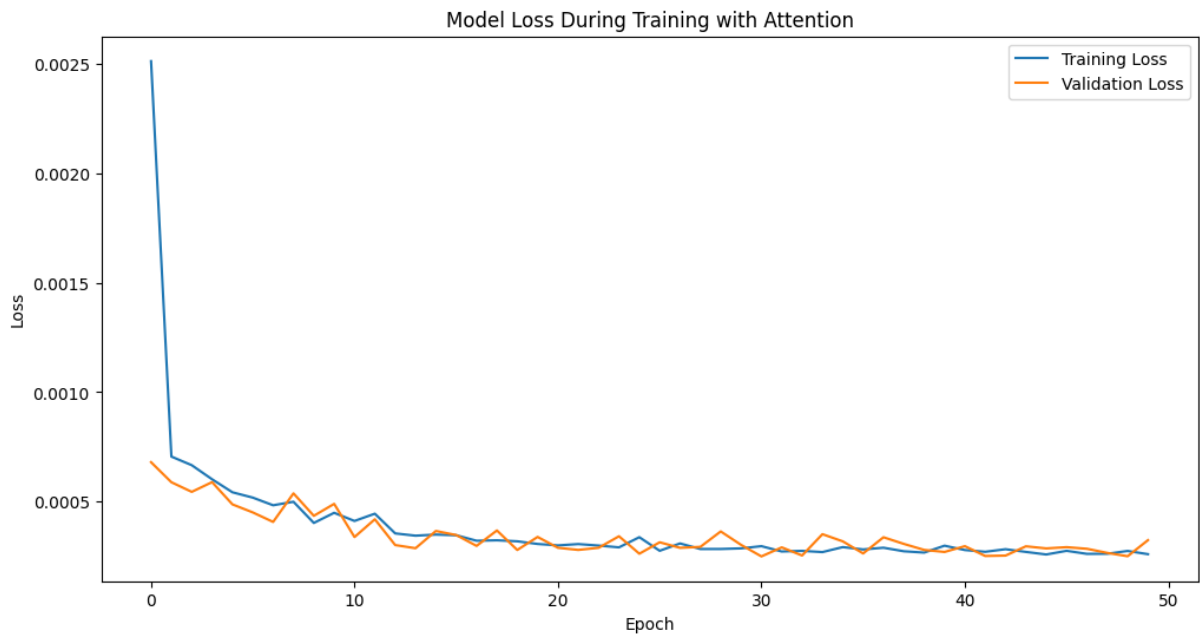


Figure 26: Training and Validation Loss of LSTM-ATTENTION model

Table 24: Evaluation metrics of LSTM-ATTENTION model

MAE	MSE	RMSE	MAPE	Accuracy
0.1482	0.0424	0.0624	5.4161 %	99%

5 Discussion

The research consists of examining six different forecasting models and analyzing their performance based on MAE, MSE, RMSE, MAPE, and accuracy. The simple design in this case is the baseline ANN-LSTM which demonstrates quite a lot of error rates but recorded 97% accuracy rate. Nonetheless, this performance is not as good as more advanced versions of the model. The introduction of Fuzzy logic also helps in performance gains by lowering both MAE and MAPE, leading to an increase in accuracy to 98%. Causing less misclassification with the use of cross-entropy helps in the classification accuracy which is now at 99%, but the error minimization has been compromised slightly when compared with fuzzy logic system. fuzzy logic application along with the LSTM model does not perform well since the error rates are high, on the other hand, LSTM-ATTENTION tends to perform the best even among LSTM variants. It has the lowest MAE (7.8652) and MAPE (4.3401%) at an accuracy level of 99%. This shows how attention mechanisms help in filtering out the necessary inputs with ease thus improving accuracy and lowering the error rate.

Table 25: Comparison of Forecasting Models and Their Evaluation Metrics for the Ethereum Prices.

Hybrid model	MAE	MSE	RMSE	MAPE	Accuracy
ANN-LSTM	23.8088	754.9796	27.4768	12.2684%	97%
ANN-LSTM with Fuzzy logic	8.6202	167.3954	12.9381	4.8461%	98%
ANN-LSTM with Cross entropy	9.7276	175.4941	13.2474	5.3993%	99%
LSTM with fuzzy logic	488.8095	240424.6334	490.3311	283.9285%	96%
LSTM with cross-entropy	18.1591	483.5446	21.9896	9.78781%	97%
LSTM-ATTENTION	7.8652	150.2508	12.2576	4.3401%	99%

Table 26 compares the results obtained from the analysis of the six forecasting models. The baseline ANN-LSTM model does well with an average deviation absolute of 0.1808, a mean squared error of 0.0717, and an accuracy of 99%; however, its mean absolute percentage error is a bit high at 5.4972%. Both the ANN-LSTM models with cross-entropy and fuzzy logic improve on this with lower MAE and MAPE values, which means more accurate predictions. The ANN-LSTM with fuzzy logic achieves the best balance with an MAE of 0.1537 and the minimum MAPE of 4.5420%. The results obviously demonstrate that the LSTM-ATTENTION architecture outperforms all the remaining models. The LSTM-ATTENTION fetches much lower error indices and has the least MSE (0.0424) and RMSE (0.0624), which means it is better at managing prediction errors than other models.

Table 26: Comparison of Forecasting Models and Their Evaluation Metrics for the Natural Gas Prices.

Hybrid model	MAE	MSE	RMSE	MAPE	Accuracy
ANN-LSTM	0.1808	0.0717	0.2678	5.4972%	99%
ANN-LSTM with Cross entropy	0.1598	0.0606	0.2462	4.5495%	99%
ANN-LSTM with Fuzzy logic	0.1537	0.0530	0.2303	4.5420%	99%
LSTM-ATTENTION	0.1482	0.0424	0.0624	5.4161 %	99%
LSTM with cross-entropy	1.6359	4.4070	2.0992	28.7822 %	99%
LSTM with fuzzy logic	139.1647	21551.5044	146.8043	4277.1636%	95%

6 Conclusions

The purpose of our study was mainly to enhance the efficacy of prediction models, especially in the energy and financial sectors. However, given a variety of underlying market dynamics, forecasting the price of cryptocurrencies as well as gas is a delicate undertaking. Thus, in contrast to classical approaches like time series analysis or econometric modeling,

which may take considerably longer to develop appropriate models, advances in deep learning and artificial intelligence have recently enabled the development of better and more precise prediction models. Hybridization can increase the precision of price forecasts by utilizing the two modeling systems in different ways.

Initially, as reported earlier, we integrated ANNs and LSTMs adopting fuzzy logic and cross-entropy, which augmented the predictions. In other aspects, we integrated LSTM with fuzzy logic and cross-entropy and lastly leaped into LSTM using the Attention Mechanism in each dataset. The results indicate that the LSTM-ATTENTION model is the most effective among all other models across both data sets. In the first data set, it records the best error metrics, with an MAE of 7.8652, 4.3401% MAPE, and 99% accuracy. Also, in regards to the second data set, the LSTM-ATTENTION model performed best in error control, as exhibited in the MSE (0.0424) and RMSE (0.0624), making it the best model for predicting error management. This shows again that the attention mechanism has reinforced benefits if implemented in the LSTM model, optimally improving the forecasting ability.

Our findings therefore have significant implications for investors, traders, and politicians, who all base their educated decisions on precise forecasting. It is worth noting that our models performed well, but no model should be expected to be accurate. In our future work, we may think about adding wavelet decomposition or empirical wavelet decomposition (EWD) to the strategies we developed for enhancing our prediction models. These methods can perform time series data analysis over several frequency levels, which allows a higher level of detail in the search for concealed characteristics and behavior of the time series data.

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