Expectations Hypothesis Revisited

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Abstract

In this paper we study the three academically prevalent versions of the Log form of the Expectations Hypothesis (LEH) for the long-term zero-coupon treasury bond yields using the level variables and find clear affirmation for one version, general affirmation for the second version and clear negation for the third version. These results validate the LEH theory while explaining the reasons behind the widespread negative empirical evidence reported in the past. We also develop two more models for estimating and forecasting the bond yields using a simple average index of the yields for different maturities.

Keywords: Zero coupon bonds; Bond yields; Expectation Hypothesis; Estimation; Forecasting.
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1. Introduction.

The Expectations Hypothesis (EH) is the simplest and the most fundamental model of the term structure of the bond yields that advocates forecasting of long-term (short-term) bond returns based on expectations formed from current short-term (long-term) bond yields. This rationale is based on the theoretical equivalence of a buy and hold strategy with that of a roll-over strategy, whereby the investor can either buy a long-term bond and hold it to its maturity or buy a short-term bond and reinvest its proceeds in a subsequent short term bond and keep repeating this in a roll-over manner over the lifetime of the long term bond. Understanding the term structure of bond yields is important for portfolio hedging by investors, making future investment decisions by firms and formulating monetary and debt policies by the policy makers.

The extant literature on EH gained prominence since Macaulay (1938), Hicks (1939) and Lutz (1940) and has been active ever since. Initially, the EH was merely hypothesized and not formally based on any fully specified equilibrium model and as a result it became associated with not just one but rather a collection of mathematical statements. However, it was empirically tested much later with two classical approaches adopted by Fama and Bliss (1987) and Campbell and Shiller (1991) respectively, with both rejecting it. The subsequent literature is generally derived from either of these two approaches with additional macroeconomic factors taken into consideration (Backus et al., 2001; Dai and Singleton, 2002; Cochrane and Piazzesi, 2005; Sangvinatsos, 2008; Cieslak and Povala, 2010; Duffee, 2011; Joslin et al., 2014; Abrahams et al., 2016; Aruoba, 2020 etc.).

In this paper, we argue that the extant contrary empirical evidence is due to the use of yield spreads in the regression models testing the EH, which generates results different from those of direct regressions of the level variables considered in the EH equations. Based on our empirical validation of the EH, we further present two very simple models for forecasting bond yields that are based on an equally-weighted average index of the yield rates for different maturities similar to the stock market index and find that the R^2 values of these regressions are as high as 65% to 85% which are much higher than those reported in earlier studies. We also conduct unit root and cointegration tests to demonstrate the statistical validity of these regressions in face of stationarity concerns.

The rest of the paper is organized as follows: Section 2 briefly explains the underlying theory and the fresh evidence from this study, Section 3 discusses the data and the empirical results obtained for the various tests with Section 4 summarizing and concluding the paper.

2. Theory in brief.

The literature on the term structure distinguishes between the Pure Expectations Hypothesis (PEH), which says that expected excess returns on long-term over short-term bonds are zero, and the Expectations Hypothesis (EH), which says that the expected excess returns are constant over time, the terminologies being attributed to Lutz (1940).

To start with, the relationship between the price of a bond going to mature '*n*' periods later (i.e. $P_{n,t}$) at time '*t*' and its yield (i.e. $Y_{n,t}$) is given by:

$$P_{n,t} = 1/(1 + Y_{n,t})^n \qquad (1)$$

Assuming that zero-coupon bond prices can be observed or estimated from coupon bond prices, Campbell, Lo and MacKinlay (1997) document three different types of PEH according to the time horizons being considered. The first form of the PEH equates the returns from a one-period maturity and n-period maturity bonds over the next one time period:

$$(1 + Y_{1,t}) = E_t[1 + R_{n,t+1}] = E_t[P_{n-1,t+1}/P_{n,t}] \qquad (2)$$

Here, $Y_{1,t}$ is the simple yield of the one-period bond at time '*t*', while $P_{n,t}$ and $P_{n-1,t+1}$ are the prices of an n-period bond at time '*t*' and '*t+1*' respectively. The second form of the PEH equates the returns from a one-period maturity and n-period maturity bonds over the next 'n' time periods (thus implying the equivalence of return from holding an n-period bond to maturity with that from rolling over successive investments in a one-period bond over the next 'n' time periods):

$$(1 + Y_{n,t})^n = E_t[(1 + Y_{1,t})(1 + Y_{1,t+1}) \ldots (1 + Y_{1,t+n-1})] \qquad (3)$$

The third form of the PEH derives from the second form of the PEH, whereby the return from the instantaneous forward rate at (n-1)-period, denoted by $F_{n-1,t}$, equals the expected return at (n-1)-period ahead spot rate, denoted by $Y_{1,t+n-1}$ as follows:

$$(1 + F_{n-1,t}) = E_t[(1 + Y_{1,t+n-1})] \qquad (4)$$

The empirical literature, however, uses the log forms of the above three equations (2), (3) and (4) for the sake of ease in computation, which are called the Log Pure Expectations Hypotheses (LPEH) as given below:

LPEH 1: $y_{1,t} = E_t[r_{n,t+1}]$ (5)

LPEH 2: $y_{n,t} = (1/n) \sum E_t[y_{1,t+i}]$, *for i = 0 to n-1* (6)

LPEH 3: $f_{n-1,t} = E_t[y_{1,t+n-1}]$ (7)

In the LPEH equations (5), (6) and (7), we use smaller case notations to denote the log forms of the variables used in equations (2), (3) and (4) respectively. Theory says that if any of the equations (5), (6) or (7) hold for all '*n*' and '*t*', then the other equations also hold for all '*n*' and '*t*', even though these three equations are not generally equivalent for any particular '*n*' and '*t*'. In this paper, we convert the LPEH equations (5) to (7) to Log form of Expectation Hypothesis (LEH) by considering a constant term for each of these equations and then directly test them empirically using the following regression equations:

LEH 1: $r_{n,t+1} = \alpha + \beta\, y_{1,t} + \varepsilon_{t+1}$ (8)

LEH 2: $[(1/n)\sum(y_{1,t+i})] = \alpha + \beta\; y_{n,t} + \varepsilon_{t+1}$, *for i = 0 to n-1* (9)

LEH 3: $y_{1,t+n-1} = \alpha + \beta\, f_{n-1,t} + \varepsilon_{t+1}$ (10)

We have tested the above three LEH equations using four data samples collected from different sources as well as from 1000 simulated samples of each of these original samples using Monte Carlo technique.

The extant literature further converts the above equations of yield levels to equations of yield spreads due to concerns of stationarity of the data series. For this, generally models using yield spreads of the form $(r_{n,t+1} - y_{1,t})$ or of the form $(y_{n,t} - y_{m,t})$, where m<n, are used. Fama and Bliss (1987) regress their model using the former version of the yield spread, while Campbell and Shiller (1991) use the latter version. Most of the existing empirical literature is built upon these two models. These models are as given below:

Fama-Bliss regression: $(r_{n,t+1} - y_{1,t}) = a_1 + b_1\,(f_{n-1,t} - y_{1,t}) + e_{1t+1}$ (11)

Campbell-Shiller regression: $(y_{n-m,t+m} - y_{n,t}) = a_2 + b_2\{m/(n-m)\}(y_{n,t} - y_{m,t}) + e_{2t+m}$, *(for m<n)* (12)

In the Fama-Bliss regression equation (11), substituting LPEH 1, i.e. equation (5) on the left hand side shows that $(r_{n,t+1} - y_{1,t}) = 0$, but the right hand side expression $(f_{n-1,t} - y_{1,t})$ is unknown. The Fama-Bliss regression model equates these two expressions and it is expected that $b_1 = 0$ since the left hand side expression equals zero.

Next, the Campbell-Shiller regression equation (12) can be shown to have been derived from the definition of the one-period return for an n-period bond, i.e. $R_{n,t+1}$ and equations (1) and (5). By taking logs of equation (1) and (2) and then substituting equation (1) in (2), we have:

$$r_{n,t+1} = p_{n-1,t+1} - p_{n,t} = n\, y_{n,t} - (n-1)\, y_{n-1,t+1} = y_{n,t} - (n-1)\,(y_{n-1,t+1} - y_{n,t}) \quad (13)$$

Substituting equation (13) in equation (5) and rearranging, we have:

$$(y_{n,t} - y_{1,t}) = \{(n-1)/1\}((E_t[(y_{n-1,t+1}] - y_{n,t}) \qquad (14)$$

Substituting a generalized short period 'm' (where, $1 \leq m < n$) in place of the specified short time-period of one in equation (14) and considering a constant, we get the Campbell-Shiller regression equation (12). As a result, it is expected that $b_2 = 1$.

Thus, the Fama-Bliss and Campbell-Shiller models (11) and (12) are derived from LPEH 1, i.e. equation (5), but they actually test LEH 1, i.e. equation (8) since they both consider regression constants. However, the empirical literature has rejected the LEH because the evidence says that both $b_1 \neq 0$ as well as $b_2 \neq 1$. We investigate this discrepancy between theory and evidence by testing three variations of the Fama-Bliss regression, whereby we split the main equation into three parts equating each of the three different constituent variables with each other as follows:

$$r_{n,t+1} = a_3 + b_3 f_{n-1,t} + e_{3t+1} \qquad (15a)$$

$$r_{n,t+1} = a_4 + b_4 y_{1,t} + e_{4t+1} \qquad (15b)$$

$$f_{n-1,t} = a_5 + b_5 y_{1,t} + e_{5t+1} \qquad (15c)$$

The above equation (15a) tests the ability of the instantaneous forward rate $f_{n-1,t}$ for predicting the expected return from holding an 'n' period bond over the next one time-period. For this, we compute the values of $f_{n-1,t}$ as follows:

$$f_{n-1,t} = p_{n-1,t} - p_{n,t} \qquad (16)$$

Similarly, equations (15b) and (15c) test the relationships between $r_{n,t+1}$ and $y_{1,t}$ and $f_{n-1,t}$ and $y_{1,t}$ respectively. Of these, equation (15b) is the same as LEH 1, i.e. equation (8). These split regressions reveal the reason behind the contrary results from the Fama-Bliss regression.

We also test a variant of the Campbell-Shiller regression as well, where we simply rearrange the model to convert the yield spread on the left hand side to just the level variable as follows:

$$(y_{n-m,t+m}) = a_6 + b_6 [\{m/(n-m)\}(y_{n,t} - y_{m,t}) + y_{n,t}] + e_{6t+m}, \text{ (for } m<n) \qquad (17)$$

This study finds that the LEH holds true in general even though the extant empirical evidence rejects it because the extant empirical models testing the theory are all based on regression of yield spreads and do not test the theory directly using yield levels.

The reason behind the difference in the results obtained from the yield spreads and from the yield levels is mainly arithmetic but reveals an economic truth as well. It can be explained by a simple hypothetical example where we consider two economic variables y_t and x_t such that they are statistically equal to each other i.e. $E[y_t] = E[x_t]$. However, if we consider a third variable z_t to test the relationship, it may not hold true because $E[y_t/z_t]$ may not equal $E[x_t/z_t]$. Here, we have used the ratios y_t/z_t and x_t/z_t to explain the arithmetic reason behind the difference in results from the traditional and the level regressions because the difference in logs of variables y_t and z_t is equivalent to the ratio y_t/z_t. The arithmetic reason behind the difference is that (y_t/z_t) is a completely different variable from y_t itself and hence the economic relationship between y_t and x_t is not necessarily equivalent to the relationship between (y_t/z_t) and (x_t/z_t). This can be illustrated further by considering $x_t = \{x_1, x_2\}$; $y_t = \{y_1, y_2\}$; and $z_t = \{z_1, z_2\}$, where z_1 and z_2 are non-zero values. Then, the means of the two variables y_t and (y_t/z_t) are not equal because $(y_1+y_2)/2 \neq \{(y_1/z_1)+(y_2/z_2)\}/2$. Similarly, $(x_1+x_2)/2 \neq \{(x_1/z_1)+(x_2/z_2)\}/2$. Thus, the covariance between y_t and x_t would be different from that between (y_t/z_t) and (x_t/z_t). Thus, the regression slope between y_t and x_t would be different from that between (y_t/z_t) and (x_t/z_t).

We further introduce two models, for estimating and forecasting the yield rates, based on a yield index that is a simple average of the yields of the bonds having maturities of 1 to '*n*' periods. These models aim to fill the gap left by the absence of any extant empirically valid model and are conceptually based on the stock market index, whereby the prices of the constituent stocks of an average index are correlated with the index price. The models are:

$y_{I,t} = (1/n) \sum y_{i,t},$ for $i = 1$ to n *(18)*

Index Model 1: $y_{n,t} = \alpha_1 + \beta_1 y_{I,t} + \varepsilon_{1n,t}$ *(19)*

Index Model 2: $y_{n,t} = \alpha_2 + \beta_2 \{(y_{I,t}/ y_{I,t-1/12}) y_{n,t-1/12}\} + \varepsilon_{2n,t}$ *(20)*

The Index Model 1 (IM1) estimates the yield rates based on the index rate, while the Index Model 2 (IM2) estimates them on the basis of change in the index rate multiplied with the previous value of the yield rate. Both these approaches have previously been used for estimating stock returns (Sharpe, 1964; Chakraborty et al., 2019). We also use IM1 and IM2 to forecast the yield rates 12 periods forward (using monthly data) and compare the results with that of a simple AR(12) model as follows:

Forecasting Index Model 1: $y_{n,t+1} = \alpha_3 + \beta_3 y_{I,t} + \varepsilon_{3n,t}$ *(21)*

Forecasting Index Model 2: $y_{n,t+1} = \alpha_4 + \beta_4 \{(y_{I,t}/y_{I,t-1/12})y_{n,t-1/12}\} + \varepsilon_{4n,t}$ *(22)*

Forecasting AR(12) Model: $y_{n,t+12/12} = \alpha_5 + \beta_5 y_{n,t} + \varepsilon_{5n,t}$ *(23)*

The above equations (FIM1), (FIM2) and AR(12) each indicates the ability of the market to forecast the bond yields one year forward using monthly observations depending on whether β_3, β_4 or β_5 is equal to 1 respectively. In this paper, the regression equations (21), (22) and (23) have been used for in-sample forecasting based on the full sample and for out-of-sample forecasting based on a rolling window of the past 24 months.

We next address the concerns for stationarity of the data by testing equation (8), which establishes the validity of LEH 1 for all the samples and equations (21) and (22), which are the year-forward yield forecasting models, for unit roots and then for cointegration to demonstrate the statistical validity of these regressions. We first test the dependent and the explanatory variables for unit roots using the Augmented Dickey Fuller (ADF) test and then test for cointegration between the dependent and the explanatory variables using the following equation:

$$y_{j,t} = a + b\, x_{j,t} + e_{j,t} \qquad (24)$$

where, $y_{j,t}$ is the dependent variable and $x_{j,t}$ is the independent variable as given in equations (8), (21) and (22) and then further test the residual series $e_{j,t}$ as follows:

$$(e_{j,t} - e_{j,t-1}) = a' + b'\, e_{j,t-1} + u_{j,t} \qquad (25)$$

The asymptotic critical values for testing the significance of b' is taken from Davidson and MacKinnon (1993, Table 20.2). If b' is significant then the dependent and the explanatory variables of equation (24) are cointegrated and the regression equation is statistically valid.

3. Empirical Study.

We undertake a study of four samples of zero-coupon bond yields having maturities of one to five years each, the details and the descriptive statistics of which are given in Tables 1 and 2 respectively. These four samples have been collected from:

a) The Federal Reserve Economics Database (FRED) of Federal Reserve Bank of New York, giving smoothed yields on hypothetical treasury securities based on the methodology of Gürkaynak et al. (2007) from June, 1961 to August, 2020;
b) The Ohio State University webpage providing the replication dataset of McCulloch-Kwon portfolios used by Campbell and Shiller (1991) from January, 1947 to February, 1991[1],
c) The OpenICPSR repository providing the replication dataset of Fama-Bliss portfolios used by Cochrane and Piazzesi (2005) from June, 1952 to December, 2003; and
d) Eikon of Refinitiv for the time-period January, 1995 to August, 2020.

[1] The dataset starts from December, 1946 but we have considered it from January, 1947 in keeping with the other samples starting either from January (start of the year) or from June (mid-year).

All these samples are monthly in frequency. The results of the empirical analyses have been reported in Tables 2 to 13. First, we take a look at the descriptive statistics of Table 2, where the means increase slightly with increasing maturity for all the samples but the standard deviations decrease slightly with increasing maturity. Thus, the shorter-term yields fluctuate slightly more than the longer-term yields. We have further reported the slopes of the yield series when regressed across time '*t*' and we can see that both S1 and S4 have slightly negative time slopes that indicate slight overall decrease in yield levels across time whereas S2 and S3 have slightly positive time slopes. The time slopes of all the samples S1 to S4 are very close to 0, indicating nearly equal increase and decrease in the yield levels over the time-periods considered thus resulting in nearly same values in the end as those at the start.

Table 3a provides the results of testing the theory of LEH as given in the equations (8), (9) and (10) and it can be seen that two samples S1 (FRED data) and S4 (Eikon data) give affirmative results for all the three forms of LEH whereby, $\beta = 1$ statistically. However, S2 (McCulloch-Kwon data) validates only LEH 1 and LEH 2 but not LEH 3 while S3 (Fama-Bliss data) validates only LEH 1 but not LEH 2 and LEH 3. Thus, LEH 1 gets affirmative results from all the four samples S1 to S4; LEH 2 gets affirmative results from three out of four samples; and LEH 3 gets affirmative results from two samples. We have further tested the LEH equations using 1000 Monte Carlo simulations each from the four original data samples S1, S2, S3 and S4 as tested in Campbell and Shiller (1991), the results of which have been reported in Table 3b. The simulations of all the samples S1, S2, S3 and S4 clearly validate LEH 1 and clearly reject LEH 3, while simulations of three samples S1, S2 and S4 generally validate LEH 2. Thus, we may say that LEH 1 and LEH 2 hold true while LEH 3 does not. As LEH 3 does not hold true empirically, we may also infer that instantaneous forward rates do not have any viable forecasting ability. This goes on to show that even when one form of the LEH holds true for a sample it does not automatically mean that the other forms of the LEH would also hold true. However, these results also indicate that the LEH theory holds true in general, even though the extant empirical evidence has rejected it. The main reason behind the contrary empirical evidence reported in the past is that the various extant models testing the hypothesis are generally based on Fama-Bliss model as given in equation (11) and Campbell-Shiller model as given in equation (12) and test the LEH theory by regressing yield spreads and not the yield levels and regressing spreads don't generate the same results as regressing levels.

The results in Table 4 report the parameters for the Fama-Bliss regressions as given in equation (11) and we can see that the results are similar to those reported in the literature since $b_1 \neq 0$ for all the samples except S4. It may also be noted that b_1 keeps increasing for increasing maturity '*n*', even though the holding period of the bond is only one year just like the results reported in the literature. Similarly, the results in Table 5 report the parameters for the Campbell-Shiller regressions from equation (12) where again the results are similar to those reported in the literature whereby $b_2 \neq 1$ (for $m=1$) for S1, S2 and S3 but not for S4. For $m>1$, $b_2 = 1$ statistically (though numerically not so) for all the samples and these results are also similar to that reported in literature (Campbell and Shiller, 1991). Thus, the only exception is the Eikon sample S4, for which statistically $b_1 = 0$ and $b_2 = 1$ but even then, we can see that numerically $b_1 \neq 0$ and $b_2 \neq 1$ and the values of t-statistics are small only because of the comparatively high standard error values. The reasons behind the contrary results of the Fama-Bliss and the Campbell-Shiller regressions have already been explained earlier, the main reasons being that the Fama-Bliss model is not a valid test of the LEH theory while the Campbell-Shiller model regresses yield spreads and not yield levels and this generates contrary results. These conclusions are further borne out by the results from regressing variants of these models as given in equations (15a, 15b and 15c) and (17), which have been reported in Tables 6 and 7 respectively. The results of equation (15a) as given in Table 6 are mixed and equally divided since two samples S1 and S2 show that $b_3 = 1$ while the other two samples S3 and S4 show that $b_3 \neq 1$ when we regress only the holding return $r_{n,t+1}$ and the forward rate $f_{n-1,t}$ directly. But again the results of equation (15c) are predominantly negative with three out of four samples S1, S3 and S4 showing $b_5 \neq 1$ statistically thus indicating that the forward rate $f_{n-1,t}$ cannot be forecasted by the one-year bond yields, even though the holding return $r_{n,t+1}$ can be and hence there does not exist any one-to-one empirical relationship between $r_{n,t+1}$ and $f_{n-1,t}$. Hence, the latter cannot forecast the former and the expressions $(r_{n,t+1} - y_{1,t})$ and $(f_{n-1,t} - y_{1,t})$ are not economically or empirically related. Further, it may be noted that $(r_{n,t+1} - y_{1,t})$ is not zero but a constant according to LEH 1, indicating that b_1 cannot be zero. These two reasons when coupled with the mathematical fact that regressing yield spreads is not the same as regressing yield levels explain as to why the Fama-Bliss regression is not a valid test of LEH.

We then look at the results of equation (15b) which are the same as the results for equation (8) and we can see that the results are affirmative for all the four samples (i.e. $b_4 = 1$), showing that the one-year holding returns for the bonds can be forecasted by the yields of one-year bonds.

Similarly, the results in Table 7 show that equation (17), which is just a rearranged variant of the Campbell-Shiller model, gives different slope coefficients and thus different conclusions, leading to the surmise that regressing yield spreads is not equivalent to regressing yield levels. From Table 7, we can see that the coefficient $b_4 = 1$ statistically for the three samples S1, S2 and S4 and for the remaining sample S3, it holds true for $m=1$. This further confirms LEH 1 since the Campbell-Shiller model is based on this version of LEH. It may also be noted that the values of the adjusted R^2 are very small and sometimes negative for the Fama-Bliss and the Campbell-Shiller models that regress yield spreads and are much higher for the other equations that regress the level variables. The standard errors are all Newey-West corrected for heteroskedasticity and autocorrelation wherever there are overlapping data due to lags between the dependent and the explanatory variables. Though the some studies have considered higher number of lags for Newey West correction (for example Cochrane and Piazzesi, 2005 consider 18 lags instead of 12 for one year lag using monthly data), we have conservatively used 12 lags only for each year difference in forecasting horizon because LEH 2, LEH 3 and the Campbell Shiller models studied here consider up to 4 years difference between the dependent and the explanatory variables resulting in up to 48 lags in Newey West corrections. We feel that this provides conservative scenario of the analyses because increasing the number of lags increase the standard error values which make it even more likely to not reject the null hypothesis of the beta equaling 1.

We now refer to Tables 8a and 8b which report the results for the Index Models 1 and 2 (IM1 and IM2) as given in equations (19) and (20) for estimating yields using a simple equally-weighted index constructed from the average of the yields of bonds with different maturities. The first model simply uses the index rate while the second model uses the change in the index rate multiplied by the previous yield value as the explanatory variable. This approach is similar to that used in estimating stock returns. As can be seen from Table 8a, the values of slope coefficient β_1 of IM1 are all significantly different from 1 for all the four samples, even though they are numerically very close to 1. This drawback is overcome in IM2, where the values of the slope coefficient β_2 are both numerically as well as statistically equal to 1 for all the four samples. The accuracy of IM2 is also better than that of IM1 as can be seen from Table 9, where we can see that the correlations of IM2 are higher and the Mean Squared Errors (MSEs) are lower than those of IM1.

We use the index models for both in-sample and out-of-sample forecasting purposes as well. The equations (21) and (22), which are called FIM1 and FIM2, are used for both in-sample forecasting using the full sample and out-of-sample forecasting using a rolling window estimation of the regression parameters from the past 24 months. We also compare the results of FIM1 and FIM2 with that of a simple AR(12) model and find their characteristics quite similar as can be seen from Tables 10a, 10b and 10c. However, the Index Models have an advantage in the sense that they are based on a common index and are more convenient to use than using different data series for different yield rates as is required in the AR(12) model. From the Tables 10a, 10b and 10c, we can see that the values of the slope coefficients β_3, β_4 and β_5 are quite similar and are both numerically and statistically close to 1 for three samples S1, S2 and S3, though for S4 they are statistically different from 1. We also refer to the results of equation (17), which is a variant of the Campbell-Shiller model, given in Table 7 and see that the slope coefficients $b_6 =1$ statistically for $m=1$ (i.e. for one-year forward forecasting) for all the samples and hence study the performance of this model further to compare it with those of FIM1, FIM2 and AR(12). The performance of these four models have been reported in Table 11a and we can see that equation (17) has mostly the highest values for MSEs for all the four samples S1, S2, S3 and S4 and hence we discard it in favor of the more accurate index models FIM1 and FIM2. Of these two models, we can find that for the three samples S1, S2 and S3, the FIM2 has smaller MSEs for maturities of 1 and 2 years but FIM1 has smaller MSEs for maturities of 3, 4 and 5 years. However, this pattern is just reversed for the fourth sample S4, where FIM1 has smaller MSEs for maturities of 1 and 2 years and FIM2 has smaller MSEs for maturities of 3, 4 and 5 years. Thus, we feel that both FIM1 and FIM2 are useful models for in-sample forecasting one-year ahead yield values for bonds of different maturities. Likewise, from Table 11b, we can see that the results for out-of-sample forecasting using AR(12), FIM1 and FIM2 also gives similar results as for in-sample forecasting. However, all the MSE values for out-of-sample forecasting are lower because here we use a rolling window of the past 24 months for estimating the parameters (the constant and the slope) used for forecasting, which gives more accurate results than using the full sample as is done for in-sample forecasting.

We next test the equations (8), (21) and (22) for unit roots and cointegration to demonstrate the validity of these regressions in spite of using level variables in these equations. The results are given in Tables 12 and 13. From Table 12 we can see that the data series of both dependent and explanatory variables for equations (21) and (22) are non-stationary for all the four samples, whereas for equation (8), only the explanatory variable data series are non-stationary for S1, S2 and S3 while both dependent and explanatory variable data series are non-stationary for S4. Next, Table 13 gives the results from the cointegration tests and it can be seen that the dependent and explanatory variables are cointegrated for all the three equations (8), (21) and (22), (i.e. LEH 1, FIM1 and FIM2) for three samples S1, S2 and S3 though for S4, the cointegration exists only for equation (8), i.e. LEH 1. These results indicate that these models are statistically valid in general.

4. Summary and Conclusions.

The extant empirical literature rejects the LEH based on tests of regression that use yield spreads (Fama and Bliss, 1987; Campbell and Shiller, 1991; Cochrane and Piazzesi, 2005 etc.). This study tests the three versions of LEH traditionally stated in the literature, using level variables directly using both actual and simulated data and finds affirmative results validating the theory in general. We have further tested the regression equations of the yield levels for cointegration and report that these regression equations are statistically valid. This is an important finding that holds both academic as well as practical significance. We further develop two models for estimating yields of bonds of different maturities using an equally-weighted average index very much similar to the stock index. These models can be extended for both in-sample and out-of-sample forecasting of yield values using monthly data with reasonable accuracy while being very convenient to use. The efficacy of these models is in keeping with the Expectations Hypothesis, which basically says that future returns or long-term (short-term) bond yields can be forecasted reasonably through expectations formed from current short-term (long-term) yields. We hope that these findings will answer many academic and policy-related questions.

Conflict of Interest: The authors state that they have no conflict of interest.

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Table 1: Sample names and their data description:

S.No.	Source of Data	Sample Name	From	To	Frequency	Coupon Rate	Years to Maturity	Observations
1	Gürkaynak et al. (2007) fitted data from Federal Reserve Economics Database (FRED)	S1	June 1961	August 2020	Monthly	Zero	1,2,3,4 and 5	711
2	McCulloch-Kwon sample used by Campbell and Shiller (1991)	S2	January 1947	February 1991	Monthly	Zero	1,2,3,4 and 5	530
3	Fama-Bliss sample used by Cochrane and Piazzesi (2005)	S3	June 1952	December 2003	Monthly	Zero	1,2,3,4 and 5	619
4	Eikon (of Refinitiv)	S4	January 1995	August 2020	Monthly	Zero	1,2,3,4 and 5	308

Table 2: Descriptive statistics showing mean yield, standard deviation, slope with time, its t-statistics and the constant over time:

$$y_{n,t} = \alpha + \beta\, t + \varepsilon_{n,t}$$

S.No.	Sample Name	Year	Mean	Std Dev	β	t-stat	α
1	S1	1	4.97	3.31	-8.73E-03	-17.15	8.08
		2	5.18	3.26	-8.71E-03	-17.51	8.28
		3	5.36	3.18	-8.49E-03	-17.46	8.38
		4	5.51	3.11	-8.20E-03	-17.18	8.43
		5	5.64	3.04	-7.89E-03	-16.78	8.44
2	S2	1	5.53	3.35	1.83E-02	34.78	0.69
		2	5.72	3.33	1.86E-02	38.35	0.77
		3	5.83	3.30	1.87E-02	40.43	0.86
		4	5.91	3.28	1.88E-02	41.67	0.93
		5	5.98	3.26	1.88E-02	42.66	1.00
3	S3	1	5.67	2.91	4.61E-03	7.35	4.24
		2	5.87	2.86	5.20E-03	8.54	4.26
		3	6.04	2.79	5.52E-03	9.39	4.33
		4	6.17	2.76	5.84E-03	10.16	4.36
		5	6.25	2.72	5.99E-03	10.62	4.39
4	S4	1	2.59	2.22	-1.89E-02	-20.26	5.50
		2	2.79	2.17	-1.93E-02	-22.81	5.77
		3	3.00	2.08	-1.93E-02	-25.64	5.98
		4	3.20	1.99	-1.90E-02	-28.47	6.13
		5	3.39	1.90	-1.86E-02	-31.20	6.27

Table 3a: Results for regressions testing the three versions of the Log Expectations Hypothesis (LEH) using equations (8), (9) and (10)

LEH 1: $r_{n,t+1} = \alpha + \beta\, y_{1,t} + \varepsilon_{t+1}$

LEH 1 Years	S1					S2					S3					S4				
	β	SE	t-stats	α	Adj R^2	β	SE	t-stats	α	Adj R^2	β	SE	t-stats	α	Adj R^2	β	SE	t-stats	α	Adj R^2
2	1.06	0.073	0.78	0.19	81.77	1.11	0.088	1.21	-0.38	82.92	1.11	0.095	1.17	-0.22	76.62	1.12	0.075	1.58	0.30	83.04
3	1.04	0.132	0.31	0.63	56.30	1.15	0.161	0.94	-0.62	61.81	1.13	0.178	0.75	-0.05	49.94	1.13	0.140	0.96	0.94	56.35
4	1.00	0.185	0.01	1.13	37.63	1.18	0.223	0.83	-0.85	47.15	1.17	0.250	0.68	-0.13	35.25	1.10	0.193	0.50	1.68	36.62
5	0.95	0.234	0.19	1.62	25.29	1.21	0.285	0.74	-1.07	36.57	1.18	0.306	0.59	-0.23	26.51	1.06	0.246	0.23	2.34	23.81

LEH 2: $[(1/n)\sum(y_{1,t+i})] = \alpha + \beta\ y_{n,t} + \varepsilon_{t+1},\ \ for\ i = 0\ to\ n\text{-}1$

LEH 2 Years	S1					S2					S3					S4				
	β	SE	t-stats	α	Adj R^2	β	SE	t-stats	α	Adj R^2	β	SE	t-stats	α	Adj R^2	β	SE	t-stats	α	Adj R^2
2	0.96	0.036	1.00	-0.05	93.55	0.94	0.043	1.45	0.25	93.33	0.93	0.047	1.52	0.21	90.10	0.94	0.037	1.72	-0.13	92.93
3	0.92	0.062	1.21	-0.01	85.56	0.88	0.072	1.62	0.54	86.32	0.85	0.073	2.06	0.58	78.68	0.86	0.072	1.90	-0.19	81.47
4	0.90	0.083	1.21	-0.02	79.49	0.84	0.095	1.66	0.78	81.67	0.77	0.087	2.59	1.01	70.99	0.81	0.107	1.78	-0.29	73.79
5	0.88	0.102	1.13	-0.07	74.24	0.81	0.115	1.67	0.99	78.34	0.73	0.096	2.84	1.32	67.02	0.78	0.125	1.73	-0.48	70.23

LEH 3: $y_{1,t+n-1} = \alpha + \beta\, f_{n-1,t} + \varepsilon_{t+1}$

LEH 3 Years	S1					S2					S3					S4				
	β	SE	t-stats	α	Adj R^2	β	SE	t-stats	α	Adj R^2	β	SE	t-stats	α	Adj R^2	β	SE	t-stats	α	Adj R^2
2	0.91	0.071	1.24	0.01	76.14	0.52	0.130	3.72	4.11	18.58	0.00	0.130	7.67	4.79	-2.19	0.87	0.072	1.86	-0.21	72.98
3	0.84	0.110	1.46	0.12	56.46	0.40	0.170	3.55	4.75	8.94	-0.18	0.176	6.74	5.88	2.35	0.70	0.157	1.87	-0.20	39.96
4	0.82	0.145	1.22	-0.01	47.61	0.30	0.192	3.66	5.32	3.47	-0.31	0.174	7.53	6.60	12.64	0.67	0.219	1.50	-0.55	30.81
5	0.79	0.177	1.16	-0.03	38.78	0.21	0.201	3.93	5.85	0.12	-0.44	0.160	8.98	7.27	26.62	0.66	0.223	1.53	-0.93	25.26

The standard errors are Newey-West corrected for HAC (Heteroskedasticity and Auto-Correlation among the error residuals) due to overlapping data where 12 lags are considered for 12 months in each year added to forecast horizon. Thus, LEH 1 considers 12 lags, whereas LEH 2 and LEH 3 considers 12, 24, 36 and 48 lags for $y_{1,t+1}$, $y_{1,t+2}$, $y_{1,t+3}$, and $y_{1,t+4}$ forecasts respectively.

Table 3b: Results for regressions testing the three versions of the Log Expectations Hypothesis (LEH) using equations (8), (9) and (10) based on 1000 Monte Carlo samples simulated from each of the original data samples S1, S2, S3 and S4.

LEH 1: $r_{n,t+1} = \alpha + \beta\, y_{1,t} + \varepsilon_{t+1}$

Sample	$\beta=1$			
	2	3	4	5
S1	1.000	1.000	1.000	1.000
S2	1.000	1.000	1.000	1.000
S3	0.999	1.000	1.000	1.000
S4	0.999	1.000	1.000	1.000

LEH 2: $[(1/n)\sum(y_{1,t+i})] = \alpha + \beta\; y_{n,t} + \varepsilon_{t+1},\;\; for\ i = 0\ to\ n\text{-}1$

Sample	$\beta=1$			
	2	3	4	5
S1	0.727	0.987	0.998	1.000
S2	0.485	0.728	0.867	0.934
S3	0.105	0.050	0.000	0.000
S4	0.429	0.786	0.978	0.965

LEH 3: $y_{1,t+n-1} = \alpha + \beta\, f_{n-1,t} + \varepsilon_{t+1}$

Sample	$\beta=1$			
	2	3	4	5
S1	0.000	0.000	0.000	0.000
S2	0.001	0.000	0.000	0.000
S3	0.000	0.000	0.000	0.000
S4	0.000	0.000	0.000	0.000

The above tables report the fractions of 1000 simulated samples that do not reject the null hypothesis that $\beta=1$.

Table 4: Results for Fama-Bliss Regressions: $(r_{n,t+1} - y_{1,t}) = a_1 + b_1 (f_{n-1,t} - y_{1,t}) + e_{1,t+1}$

Years	S1					S2					S3					S4				
	b_1	SE	t-stats	a_1	Adj R^2	b_1	SE	t-stats	a_1	Adj R^2	b_1	SE	t-stats	a_1	Adj R^2	b_1	SE	t-stats	a_1	Adj R^2
1→2	0.76	0.268	2.83	0.14	6.39	0.92	0.295	3.12	-0.12	9.84	0.90	0.221	4.08	0.06	10.90	0.09	0.512	0.17	0.59	-4.48
2→3	0.93	0.328	2.82	0.14	6.82	1.19	0.407	2.93	-0.41	9.02	1.18	0.296	3.98	-0.10	11.33	0.32	0.527	0.61	1.02	-3.00
3→4	1.10	0.362	3.03	0.04	7.83	1.39	0.524	2.65	-0.71	7.65	1.46	0.372	3.93	-0.39	13.24	0.51	0.492	1.05	1.28	-0.86
4→5	1.26	0.383	3.28	-0.12	8.83	1.51	0.675	2.23	-0.98	6.49	1.15	0.485	2.36	-0.16	4.85	0.68	0.478	1.42	1.41	1.04

The standard errors are Newey-West corrected for HAC (Heteroskedasticity and Auto-Correlation among the error residuals) due to overlapping data where 12 lags are considered for 12 months in each year.

Table 5: Results for Campbell-Shiller Regressions: $(y_{n-m,t+m} - y_{n,t}) = a_2 + b_2\{m/(n-m)\}(y_{n,t} - y_{m,t}) + e_{2,t+m}$, *(for m<n)*

Sample S1 Years (n)	m=1					m=2					m=3					m=4				
	b_2	SE	t-stats	a_2	Adj R^2	b_2	SE	t-stats	a_2	Adj R^2	b_2	SE	t-stats	a_2	Adj R^2	b_2	SE	t-stats	a_2	Adj R^2
2	-0.52	0.54	2.83	-0.14	-0.85															
3	-0.80	0.59	3.02	-0.06	0.18	0.30	0.76	0.92	-0.57	-1.57										
4	-1.06	0.63	3.27	0.00	1.36	-0.04	0.74	1.39	-0.38	-1.93	0.78	0.57	0.38	-1.00	0.82					
5	-1.31	0.66	3.52	0.05	2.55	-0.36	0.74	1.84	-0.24	-1.44	0.33	0.60	1.12	-0.70	-1.47	0.55	0.54	0.83	-1.11	-0.47
Sample S2 Years (n)	m=1					m=2					m=3					m=4				
	b_2	SE	t-stats	a_2	Adj R^2	b_2	SE	t-stats	a_2	Adj R^2	b_2	SE	t-stats	a_2	Adj R^2	b_2	SE	t-stats	a_2	Adj R^2
2	-0.84	0.59	3.12	0.12	0.29															
3	-1.15	0.70	3.06	0.21	1.32	-0.55	1.07	1.44	0.15	-1.71										
4	-1.44	0.81	3.00	0.25	2.22	-0.83	1.22	1.50	0.29	-0.92	0.34	1.11	0.60	0.03	-2.42					
5	-1.69	0.95	2.83	0.28	2.79	-0.97	1.34	1.47	0.34	-0.64	0.10	1.16	0.78	0.21	-2.69	1.12	1.17	0.10	-0.08	0.12
Sample S3 Years (n)	m=1					m=2					m=3					m=4				
	b_2	SE	t-stats	a_2	Adj R^2	b_2	SE	t-stats	a_2	Adj R^2	b_2	SE	t-stats	a_2	Adj R^2	b_2	SE	t-stats	a_2	Adj R^2
2	-0.80	0.44	4.08	-0.06	0.71															
3	-1.19	0.54	4.06	0.04	2.43	-0.17	0.72	1.62	-0.31	-2.12										
4	-1.60	0.61	4.28	0.14	4.87	-0.61	0.80	2.00	-0.10	-1.01	0.21	0.49	1.63	-0.49	-2.10					
5	-1.59	0.69	3.73	0.16	3.64	-0.73	0.97	1.79	0.01	-0.79	0.26	0.69	1.07	-0.35	-2.09	0.77	0.44	0.54	-0.62	0.31
Sample S4 Years (n)	m=1					m=2					m=3					m=4				
	b_2	SE	t-stats	a_2	Adj R^2	b_2	SE	t-stats	a_2	Adj R^2	b_2	SE	t-stats	a_2	Adj R^2	b_2	SE	t-stats	a_2	Adj R^2
2	0.83	1.02	0.17	-0.59	-1.54															
3	0.37	1.06	0.60	-0.51	-4.06	1.37	0.82	0.45	-1.42	5.85										
4	-0.01	0.99	1.02	-0.43	-4.61	1.04	0.79	0.05	-1.29	1.87	1.98	0.71	1.37	-2.55	27.52					
5	-0.28	0.93	1.37	-0.36	-4.30	0.72	0.80	0.35	-1.12	-1.41	1.63	0.79	0.79	-2.27	18.51	1.70	0.68	1.04	-3.04	26.98

The standard errors are Newey-West corrected for HAC (Heteroskedasticity and Auto-Correlation among the error residuals) due to overlapping data where 12 lags are considered for 12 months in each year added to forecast horizon..

Table 6: Results for the three splits of the Fama-Bliss Regression: Equations (15a, 15b and 15c)

$r_{n,t+1} = a_3 + b_3 f_{n-1,t} + e_{3t+1}$

Years	S1					S2					S3					S4				
	b_3	SE	t-stats	a_3	Adj R^2	b_3	SE	t-stats	a_3	Adj R^2	b_3	SE	t-stats	a_3	Adj R^2	b_3	SE	t-stats	a_3	Adj R^2
1→2	1.10	0.061	1.66	-0.52	83.60	1.13	0.071	1.81	-0.91	85.42	1.16	0.076	2.06	-0.94	80.40	1.15	0.079	1.95	-0.26	80.97
2→3	1.18	0.122	1.48	-0.96	61.41	1.23	0.136	1.70	-1.69	67.49	1.31	0.147	2.08	-1.95	59.15	1.32	0.152	2.13	-0.71	56.80
3→4	1.25	0.185	1.36	-1.39	45.47	1.30	0.200	1.52	-2.33	53.99	1.41	0.204	2.01	-2.70	47.57	1.51	0.226	2.25	-1.33	41.79
4→5	1.32	0.251	1.29	-1.82	34.96	1.37	0.268	1.39	-2.93	43.86	1.43	0.288	1.48	-2.84	34.10	1.70	0.307	2.28	-2.09	33.39

$r_{n,t+1} = a_4 + b_4 y_{1,t} + e_{4t+1}$ (same as Table 3: LEH 1 but repeated here for ready reference)

Years	S1					S2					S3					S4				
	b_4	SE	t-stats	a_4	Adj R^2	b_4	SE	t-stats	a_4	Adj R^2	b_4	SE	t-stats	a_4	Adj R^2	b_4	SE	t-stats	a_4	Adj R^2
1→2	1.06	0.073	0.78	0.19	81.77	1.11	0.088	1.21	-0.38	82.92	1.11	0.095	1.17	-0.22	76.62	1.12	0.075	1.58	0.30	83.04
2→3	1.04	0.132	0.31	0.63	56.30	1.15	0.161	0.94	-0.62	61.81	1.13	0.178	0.75	-0.05	49.94	1.13	0.140	0.96	0.94	56.35
3→4	1.00	0.185	0.01	1.13	37.63	1.18	0.223	0.83	-0.85	47.15	1.17	0.250	0.68	-0.13	35.25	1.10	0.193	0.50	1.68	36.62
4→5	0.95	0.234	0.19	1.62	25.29	1.21	0.285	0.74	-1.07	36.57	1.18	0.306	0.59	-0.23	26.51	1.06	0.246	0.23	2.34	23.81

$f_{n-1,t} = a_5 + b_5 y_{1,t} + e_{5t+1}$

Years	S1					S2					S3					S4				
	b_5	SE	t-stats	a_5	Adj R^2	b_5	SE	t-stats	a_5	Adj R^2	b_5	SE	t-stats	a_5	Adj R^2	b_5	SE	t-stats	a_5	Adj R^2
1→2	0.95	0.025	1.84	0.68	96.48	0.98	0.032	0.74	0.49	96.26	0.95	0.033	1.41	0.67	93.93	0.94	0.027	2.37	0.58	95.33
2→3	0.88	0.035	3.38	1.35	91.87	0.94	0.042	1.33	0.83	93.41	0.88	0.047	2.46	1.36	87.79	0.80	0.044	4.56	1.39	85.88
3→4	0.82	0.039	4.61	1.91	87.55	0.93	0.043	1.64	1.01	91.91	0.87	0.054	2.36	1.58	82.93	0.67	0.054	6.12	2.14	74.62
4→5	0.77	0.042	5.53	2.36	83.63	0.91	0.045	1.89	1.18	90.36	0.84	0.053	3.02	1.77	80.77	0.58	0.061	6.89	2.71	64.25

The standard errors are Newey-West corrected for HAC (Heteroskedasticity and Auto-Correlation among the error residuals) due to overlapping data where 12 lags are considered for 12 months in each year.

Table 7: Results for a variant of the Campbell-Shiller Regression: Equation (17) - $(y_{n-m,t+m}) = a_6 + b_6\,[\{m/(n-m)\}(y_{n,t} - y_{m,t}) + y_{n,t}] + e_{6t+m}$

Sample S1 Years (n)	m=1					m=2					m=3					m=4				
	b_6	SE	t-stats	a_6	Adj R^2	b_6	SE	t-stats	a_6	Adj R^2	b_6	SE	t-stats	a_6	Adj R^2	b_6	SE	t-stats	a_6	Adj R^2
2	0.91	0.07	1.24	0.01	76.14															
3	0.94	0.07	0.89	-0.08	79.27	0.84	0.11	1.46	0.12	56.46										
4	0.95	0.07	0.70	-0.12	81.17	0.88	0.11	1.07	-0.03	61.56	0.82	0.15	1.22	-0.01	47.61					
5	0.96	0.06	0.60	-0.12	82.39	0.91	0.11	0.88	-0.09	64.71	0.87	0.14	0.94	-0.14	52.11	0.79	0.18	1.16	-0.03	38.78

Sample S2 Years (n)	m=1					m=2					m=3					m=4				
	b_6	SE	t-stats	a_6	Adj R^2	b_6	SE	t-stats	a_6	Adj R^2	b_6	SE	t-stats	a_6	Adj R^2	b_6	SE	t-stats	a_6	Adj R^2
2	0.86	0.08	1.69	0.62	74.88															
3	0.89	0.08	1.48	0.57	79.55	0.76	0.12	2.03	1.22	57.86										
4	0.90	0.07	1.39	0.55	82.45	0.80	0.11	1.78	1.12	64.84	0.72	0.15	1.84	1.56	51.99					
5	0.91	0.07	1.31	0.53	84.10	0.82	0.11	1.64	1.07	68.99	0.76	0.15	1.65	1.48	58.76	0.68	0.19	1.69	1.90	46.89

Sample S3 Years (n)	m=1					m=2					m=3					m=4				
	b_6	SE	t-stats	a_6	Adj R^2	b_6	SE	t-stats	a_6	Adj R^2	b_6	SE	t-stats	a_6	Adj R^2	b_6	SE	t-stats	a_6	Adj R^2
2	0.83	0.09	1.90	0.63	64.78															
3	0.86	0.08	1.63	0.51	69.17	0.68	0.11	2.93	1.39	40.41										
4	0.86	0.08	1.74	0.59	72.58	0.72	0.11	2.58	1.28	48.08	0.58	0.12	3.49	2.05	31.34					
5	0.88	0.07	1.61	0.56	76.00	0.75	0.11	2.36	1.25	54.16	0.65	0.12	2.82	1.80	39.64	0.54	0.14	3.36	2.39	26.13

Sample S4 Years (n)	m=1					m=2					m=3					m=4				
	b_6	SE	t-stats	a_6	Adj R^2	b_6	SE	t-stats	a_6	Adj R^2	b_6	SE	t-stats	a_6	Adj R^2	b_6	SE	t-stats	a_6	Adj R^2
2	0.87	0.07	1.86	-0.21	72.98															
3	0.90	0.07	1.45	-0.31	73.23	0.70	0.16	1.87	-0.20	39.96										
4	0.91	0.07	1.25	-0.34	74.02	0.76	0.15	1.59	-0.38	44.69	0.67	0.22	1.50	-0.55	30.81					
5	0.91	0.07	1.21	-0.31	74.08	0.79	0.14	1.53	-0.42	48.51	0.72	0.21	1.33	-0.69	35.84	0.66	0.22	1.53	-0.93	25.26

The standard errors are Newey-West corrected for HAC (Heteroskedasticity and Auto-Correlation among the error residuals) due to overlapping data where 12 lags are considered for 12 months in each year added to forecast horizon.

Table 8a: Regression results for estimating bond yields using the simple average index (IM1): $y_{n,t} = \alpha_1 + \beta_1 y_{I,t} + \varepsilon_{1n,t}$

Years	S1					S2					S3					S4				
	β_1	SE	t-stats	α_1	R^2	β_1	SE	t-stats	α_1	R^2	β_1	SE	t-stats	α_1	R^2	β_1	SE	t-stats	α_1	R^2
1	1.04	0.005	7.04	-0.56	98.26	1.01	0.005	1.67	-0.31	98.49	1.03	0.007	4.11	-0.50	97.45	1.07	0.010	6.80	-0.59	97.25
2	1.03	0.002	14.49	-0.29	99.77	1.01	0.002	5.94	-0.13	99.87	1.02	0.002	9.36	-0.26	99.69	1.05	0.004	12.61	-0.35	99.52
3	1.00	0.001	4.68	0.01	99.95	1.00	0.001	0.69	0.03	99.94	1.00	0.001	1.27	0.05	99.88	1.01	0.002	5.21	-0.02	99.92
4	0.98	0.002	8.68	0.29	99.56	0.99	0.002	2.59	0.15	99.69	0.98	0.003	4.72	0.26	99.32	0.96	0.005	8.31	0.32	99.16
5	0.95	0.004	12.12	0.55	98.89	0.99	0.004	3.68	0.26	99.34	0.97	0.005	6.92	0.44	98.63	0.91	0.008	10.80	0.64	97.54

Table 8b: Regression results for estimating bond yields using the average index returns (IM2): $y_{n,t} = \alpha_2 + \beta_2 \{(y_{I,t}/y_{I,t-1/12})y_{n,t-1/12}\} + \varepsilon_{2n,t}$

Years	S1					S2					S3					S4				
	β_2	SE	t-stats	α_2	R^2	β_2	SE	t-stats	α_2	R^2	β_2	SE	t-stats	α_2	R^2	β_2	SE	t-stats	α_2	R^2
1	1.00	1.58E-03	0.35	0.00	99.82	1.00	2.22E-03	0.65	0.01	99.74	1.00	2.46E-03	0.30	0.00	99.63	1.00	2.23E-03	0.77	0.00	99.85
2	1.00	6.53E-04	0.42	0.00	99.97	1.00	9.26E-04	0.02	0.00	99.95	1.00	1.27E-03	0.02	0.00	99.90	1.00	1.37E-03	0.84	0.00	99.94
3	1.00	3.94E-04	0.09	0.00	99.99	1.00	6.90E-04	0.18	0.00	99.97	1.00	1.11E-03	0.27	0.00	99.92	1.00	1.09E-03	0.98	0.00	99.96
4	1.00	7.88E-04	0.62	0.00	99.96	1.00	1.04E-03	0.27	0.00	99.94	1.00	1.50E-03	0.61	0.01	99.86	1.00	1.41E-03	0.56	0.00	99.94
5	1.00	1.26E-03	0.93	0.01	99.89	1.00	1.55E-03	0.56	0.00	99.87	1.00	1.74E-03	1.14	0.01	99.81	1.00	2.40E-03	0.08	0.00	99.82

Table 9: Correlations and Mean Squared Errors (MSE) between actual and estimated bond yields using Index Models IM1 and IM2, i.e. equations (19) and (20):

Sample	Model	Year 1		Year 2		Year 3		Year 4		Year 5	
		Correlation (%)	MSE	Correlation (%)	MSE	Correlation (%)	MSE	Correlation (%)	MSE	Correlation (%)	MSE
S1	IM 1	99.13	0.19	99.89	0.02	99.97	0.01	99.78	0.04	99.45	0.10
	IM 2	99.91	0.02	99.98	0.00	99.99	0.00	99.98	0.00	99.94	0.01
S2	IM 1	99.24	0.17	99.94	0.01	99.97	0.01	99.84	0.03	99.67	0.07
	IM 2	99.87	0.03	99.98	0.01	99.99	0.00	99.97	0.01	99.94	0.01
S3	IM 1	98.71	0.22	99.85	0.03	99.94	0.01	99.66	0.05	99.31	0.10
	IM 2	99.81	0.03	99.95	0.01	99.96	0.01	99.93	0.01	99.91	0.01
S4	IM 1	98.56	0.14	99.71	0.03	99.96	0.00	99.57	0.03	98.78	0.09
	IM 2	99.91	0.01	99.95	0.00	99.97	0.00	99.96	0.00	99.91	0.01

Table 10a: Full-sample regression results for forecasting bond yields one year forward using the simple average index (FIM1):

$$y_{n,t+1} = \alpha_3 + \beta_3\, y_{I,t} + \varepsilon_{3n,t}$$

Years	S1					S2					S3					S4				
	β_3	SE	t-stats	α_3	R^2	β_3	SE	t-stats	α_3	R^2	β_3	SE	t-stats	α_3	R^2	β_3	SE	t-stats	α_3	R^2
1	0.94	0.070	0.93	-0.06	77.27	0.88	0.080	1.51	0.59	76.67	0.86	0.087	1.61	0.51	66.94	0.91	0.080	1.18	-0.32	72.24
2	0.94	0.061	0.95	0.12	81.16	0.90	0.071	1.42	0.66	81.72	0.88	0.077	1.57	0.60	72.80	0.89	0.073	1.50	-0.08	74.12
3	0.93	0.056	1.20	0.34	83.46	0.91	0.064	1.46	0.73	84.49	0.88	0.070	1.74	0.78	76.92	0.86	0.067	2.14	0.22	75.46
4	0.92	0.052	1.58	0.57	84.83	0.91	0.062	1.49	0.80	85.91	0.88	0.066	1.84	0.91	79.06	0.82	0.063	2.89	0.54	76.12
5	0.90	0.049	2.00	0.79	85.62	0.91	0.059	1.54	0.86	86.99	0.88	0.062	2.02	1.01	80.46	0.78	0.059	3.65	0.83	76.16

Table 10b: Full-sample regression results for forecasting bond yields one year forward using the average index returns (FIM2):

$$y_{n,t+1} = \alpha_4 + \beta_4\, \{(y_{I,t}/y_{I,t-1/12})\, y_{n,t-1/12}\} + \varepsilon_{4n,t}$$

Years	S1					S2					S3					S4				
	β_4	SE	t-stats	α_4	R^2	β_4	SE	t-stats	α_4	R^2	β_4	SE	t-stats	α_4	R^2	β_4	SE	t-stats	α_4	R^2
1	0.90	0.061	1.71	0.49	78.12	0.87	0.073	1.79	0.87	77.77	0.84	0.077	2.02	0.88	69.12	0.82	0.084	2.16	0.28	70.30
2	0.92	0.058	1.45	0.40	81.36	0.89	0.069	1.57	0.77	81.90	0.86	0.074	1.83	0.79	73.31	0.84	0.072	2.25	0.25	73.93
3	0.93	0.056	1.28	0.34	83.27	0.90	0.065	1.48	0.72	84.10	0.88	0.072	1.72	0.75	76.24	0.85	0.065	2.27	0.23	75.91
4	0.94	0.055	1.16	0.31	84.42	0.91	0.065	1.40	0.69	85.32	0.88	0.069	1.73	0.76	77.71	0.86	0.063	2.18	0.23	76.90
5	0.94	0.055	1.07	0.29	85.11	0.91	0.063	1.35	0.66	86.39	0.89	0.065	1.72	0.72	79.68	0.87	0.064	2.05	0.23	77.26

The standard errors are Newey-West corrected for HAC (Heteroskedasticity and Auto-Correlation among the error residuals) due to overlapping data where 12 lags are considered for 12 months in each year.

Table 10c: Full-sample regression results for forecasting bond yields one year forward using their current values, i.e. AR(12) equation:

$y_{n,t+12/12} = \alpha_5 + \beta_5\, y_{n,t} + \varepsilon_{5n,t}$

Years	S1					S2					S3					S4				
	β_5	SE	t-stats	α_5	R^2	β_5	SE	t-stats	α_5	R^2	β_5	SE	t-stats	α_5	R^2	β_5	SE	t-stats	α_5	R^2
1	0.90	0.060	1.74	0.49	78.28	0.87	0.071	1.83	0.87	77.85	0.84	0.075	2.08	0.89	69.27	0.82	0.083	2.17	0.27	70.84
2	0.92	0.057	1.46	0.40	81.43	0.89	0.069	1.59	0.77	81.88	0.86	0.073	1.84	0.79	73.43	0.84	0.071	2.27	0.24	74.23
3	0.93	0.056	1.27	0.34	83.30	0.90	0.066	1.48	0.72	84.11	0.88	0.072	1.71	0.75	76.27	0.85	0.065	2.30	0.24	75.94
4	0.94	0.056	1.14	0.31	84.45	0.91	0.065	1.38	0.68	85.40	0.88	0.070	1.70	0.75	77.79	0.86	0.063	2.23	0.23	76.76
5	0.94	0.056	1.04	0.29	85.18	0.92	0.064	1.31	0.65	86.51	0.89	0.066	1.66	0.71	79.77	0.87	0.063	2.09	0.23	77.12

The standard errors are Newey-West corrected for HAC (Heteroskedasticity and Auto-Correlation among the error residuals) due to overlapping data where 12 lags are considered for 12 months in each year.

Table 11a: Correlations and Mean Squared Errors (MSE) between actual and forecasted bond yields one year forward (i.e. $y_{n,t+1}$) using full sample (In-sample accuracy):

Sample	Model	Year 1		Year 2		Year 3		Year 4		Year 5	
		Correlation (%)	MSE	Correlation (%)	MSE	Correlation (%)	MSE	Correlation (%)	MSE	Correlation (%)	MSE
S1	Campbell-Shiller Variant (Equation 17)	87.51	2.59	89.25	2.18	90.29	1.89	90.95	1.68		
	AR(12) Model	88.70	2.36	90.43	1.95	91.43	1.68	92.05	1.49	92.44	1.36
	FIM 1	88.15	2.47	90.29	1.98	91.52	1.66	92.26	1.45	92.68	1.31
	FIM 2	88.61	2.38	90.39	1.96	91.42	1.68	92.04	1.49	92.40	1.36
S2	Campbell-Shiller Variant (Equation 17)	86.90	2.68	89.48	2.15	91.04	1.82	91.92	1.63		
	AR(12) Model	88.54	2.37	90.73	1.91	91.93	1.64	92.61	1.49	93.19	1.37
	FIM 1	87.90	2.49	90.65	1.93	92.13	1.60	92.88	1.44	93.44	1.32
	FIM 2	88.51	2.38	90.75	1.91	91.92	1.65	92.57	1.50	93.13	1.38
S3	Campbell-Shiller Variant (Equation 17)	80.96	2.88	83.56	2.43	85.54	2.05	87.47	1.74		
	AR(12) Model	83.62	2.51	86.02	2.09	87.62	1.77	88.46	1.62	89.55	1.43
	FIM 1	82.25	2.70	85.67	2.14	87.98	1.72	89.17	1.52	89.93	1.38
	FIM 2	83.54	2.52	85.96	2.10	87.60	1.77	88.43	1.62	89.51	1.44
S4	Campbell-Shiller Variant (Equation 17)	86.30	1.12	86.53	1.04	86.71	0.94	86.74	0.85		
	AR(12) Model	84.90	1.23	86.79	1.02	87.73	0.87	88.17	0.76	88.37	0.69
	FIM 1	85.71	1.16	86.75	1.02	87.49	0.88	87.85	0.78	87.87	0.71
	FIM 2	84.62	1.25	86.65	1.03	87.74	0.87	88.27	0.75	88.47	0.68

Table 11b: Correlations and Mean Squared Errors (MSE) between actual and forecasted bond yields one year forward (i.e. $y_{n,t+1}$) using a rolling window of the past 24 months for estimating the regression parameters of FIM1 and FIM2 (Out-of-sample accuracy):

Sample	Model	Year 1		Year 2		Year 3		Year 4		Year 5	
		Correlation (%)	MSE	Correlation (%)	MSE	Correlation (%)	MSE	Correlation (%)	MSE	Correlation (%)	MSE
S1	AR(12) Model	93.63	1.47	94.68	1.19	95.34	1.00	95.76	0.87	96.02	0.79
	FIM 1	93.74	1.47	94.71	1.19	95.34	0.99	95.74	0.86	96.00	0.77
	FIM 2	93.70	1.46	94.71	1.18	95.31	1.00	95.67	0.89	95.89	0.80
S2	AR(12) Model	92.43	1.63	93.94	1.29	94.89	1.09	95.55	0.95	96.05	0.85
	FIM 1	92.60	1.63	94.02	1.29	94.87	1.09	95.41	0.96	95.86	0.86
	FIM 2	92.55	1.61	93.98	1.29	94.85	1.09	95.44	0.97	95.89	0.88
S3	AR(12) Model	89.96	1.62	91.56	1.31	92.78	1.08	93.49	0.96	94.29	0.84
	FIM 1	90.14	1.63	91.65	1.31	92.77	1.08	93.36	0.96	94.12	0.83
	FIM 2	90.08	1.60	91.62	1.31	92.75	1.09	93.33	0.98	94.07	0.86
S4	AR(12) Model	90.41	0.86	91.19	0.69	91.12	0.60	90.80	0.54	90.49	0.50
	FIM 1	90.12	0.84	90.87	0.70	91.25	0.59	91.38	0.52	91.43	0.46
	FIM 2	90.38	0.86	91.23	0.69	91.11	0.60	90.72	0.55	90.36	0.51

Table 12: Unit root tests for dependent and explanatory variables used in equations (8), (21) and (22):

Sample	Model	Year 1		Year 2		Year 3		Year 4		Year 5	
		ADF ($y_{j.t}$)	ADF ($x_{j,t}$)	ADF ($y_{j.t}$)	ADF ($x_{j,t}$)	ADF ($y_{j.t}$)	ADF ($x_{j,t}$)	ADF ($y_{j.t}$)	ADF ($x_{j,t}$)	ADF ($y_{j.t}$)	ADF ($x_{j,t}$)
S1	LPEH 1: Equation (8)			0.02	0.35	<0.01	0.35	<0.01	0.35	<0.01	0.35
	FIM 1: Equation (21)	0.29	0.45	0.37	0.45	0.41	0.45	0.45	0.45	0.48	0.45
	FIM 2: Equation (22)	0.29	0.37	0.37	0.42	0.41	0.46	0.45	0.49	0.48	0.52
S2	LPEH 1: Equation (8)			<0.01	0.31	<0.01	0.31	<0.01	0.31	<0.01	0.31
	FIM 1: Equation (21)	0.42	0.41	0.49	0.41	0.54	0.41	0.55	0.41	0.58	0.41
	FIM 2: Equation (22)	0.42	0.36	0.49	0.39	0.54	0.41	0.55	0.43	0.58	0.46
S3	LPEH 1: Equation (8)			0.02	0.79	<0.01	0.79	<0.01	0.79	<0.01	0.79
	FIM 1: Equation (21)	0.73	0.91	0.80	0.91	0.85	0.91	0.84	0.91	0.89	0.91
	FIM 2: Equation (22)	0.73	0.82	0.80	0.90	0.85	0.91	0.84	0.91	0.89	0.93
S4	LPEH 1: Equation (8)			0.41	0.56	0.26	0.56	0.13	0.56	0.05	0.56
	FIM 1: Equation (21)	0.42	0.67	0.58	0.67	0.59	0.67	0.54	0.67	0.47	0.67
	FIM 2: Equation (22)	0.42	0.58	0.58	0.68	0.59	0.67	0.54	0.59	0.47	0.49

Table 13: Cointegration tests for equations (8), (21) and (22):

$y_{j,t} = a + b\, x_{j,t} + e_{j,t}$
$(e_{j,t} - e_{j,t-1}) = a' + b'\, e_{j,t-1} + u_{j,t}$

Sample	Model	Year 1		Year 2		Year 3		Year 4		Year 5	
		b′	t-stats	b′	t-stats	b′	t-stats	b′	t-stats	b′	t-stats
S1	LPEH 1: Equation (8)			-0.065	-4.84	-0.066	-4.87	-0.067	-4.91	-0.069	-4.98
	FIM 1: Equation (21)	-0.067	-4.89	-0.074	-5.14	-0.079	-5.32	-0.083	-5.44	-0.085	-5.51
	FIM 2: Equation (22)	-0.067	-4.86	-0.073	-5.10	-0.079	-5.33	-0.087	-5.59	-0.096	-5.88
S2	LPEH 1: Equation (8)			-0.078	-4.54	-0.075	-4.46	-0.073	-4.39	-0.071	-4.35
	FIM 1: Equation (21)	-0.082	-4.68	-0.091	-4.95	-0.097	-5.10	-0.100	-5.19	-0.103	-5.27
	FIM 2: Equation (22)	-0.087	-4.82	-0.093	-4.99	-0.096	-5.09	-0.100	-5.20	-0.106	-5.36
S3	LPEH 1: Equation (8)			-0.067	-4.57	-0.064	-4.46	-0.063	-4.44	-0.073	-4.77
	FIM 1: Equation (21)	-0.070	-4.67	-0.076	-4.87	-0.082	-5.09	-0.095	-5.48	-0.093	-5.41
	FIM 2: Equation (22)	-0.077	-4.90	-0.077	-4.93	-0.084	-5.14	-0.094	-5.45	-0.095	-5.49
S4	LPEH 1: Equation (8)			-0.028	-2.52	-0.048	-3.11	-0.067	-3.55	-0.082	-3.86
	FIM 1: Equation (21)	-0.035	-2.33	-0.045	-2.63	-0.053	-2.84	-0.059	-2.96	-0.063	-3.00
	FIM 2: Equation (22)	-0.022	-1.70	-0.037	-2.29	-0.053	-2.79	-0.070	-3.24	-0.089	-3.68

The asymptotic critical values for testing the significance of b' are:

1%: -3.90;

2.5%: -3.59;

5%: -3.34;

10%: -3.04.